Course	Number		Section
Mechanical Engineering Design	MECH 441/2		T,X
Examination	**Date**	**Time**	**No. of pages**
Midterm	October 29, 2003	14:45–16:00	2
Instructor(s)			
Dr. V. N. Latinovic Dr. Ramin Sedaghati			
Materials allowed: ☐ No ☒ Yes (Please specify)	The textbook The class notes		
Calculators allowed: ☐ No ☒ Yes	Any calculator		
Special instructions:	Attempt all problems (3).		

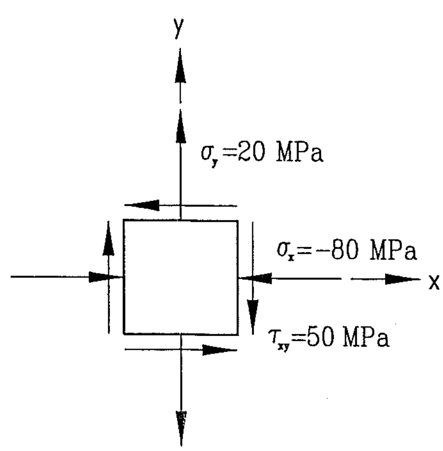
Student Name: ______________________________

I.D. No.: ______________

PROBLEM 1 (33 MARKS)

For the stress element shown in figure below, find analytically the principal stresses, and the maximum shear stresses and the planes on which theses stresses act. Draw the Mohr's circles and the principal stress element and the maximum shear stress element, both oriented properly.

Find the factors of safety against static failure using the maximum shear stress theory (Tresca theory), maximum distortion energy theory (von Mises theory) and maximum normal stress theory if material is steel with yield strength of S_y=320 MPa.

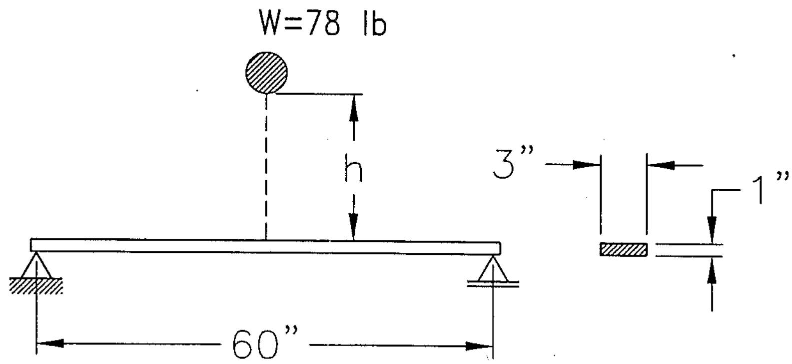


PROBLEM 2 (33 MARKS)

A simply supported steel beam is 60 inches long and has a rectangular cross section 1.0 inches wide by 3.0 inches deep. A Weight of 78 lb is dropped from a height h at mid-span.

a) If the yield strength of the material is 40000 psi, and if the beam mass is neglected, what drop height would be required to produce first evidence of yielding in the beam?

b) What is the impact factor under these circumstances?



PROBLEM 3 (34 MARKS)

For a special application, it is desired to assemble a phosphor-bronze disk to a hollow steel shaft, using an interference fit for retention. The disk is to be made of C-52100 hot-rolled phosphor bronze, and the hollow shaft is to be made of cold-drawn 1020 steel. As shown in the Figure below, the proposed nominal dimensions of the disk are 10 inches for outer diameter and 3 inches for the hole diameter, and for the shaft 3 inches outer diameter and 2 inches inner diameter. The hub length is 4 inches. The decision has been made that the maximum stress in the disk should not exceed one-half the tensile yield strength of the disk material.

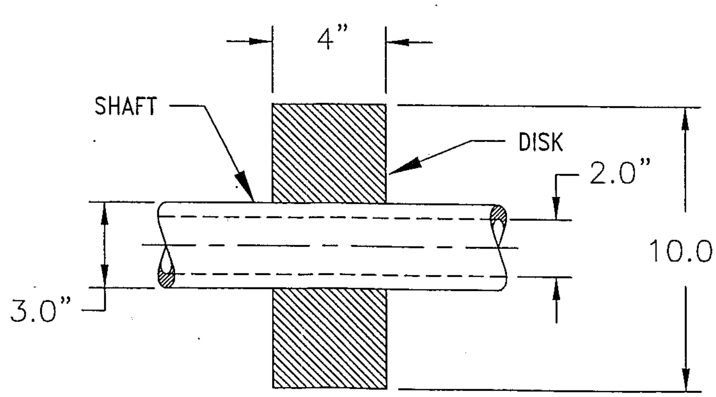
a) Based on the maximum distortion energy theory (von Mises theory) what is the maximum diametral interference that should be specified for the fit between the phosphor bronze disk and the steel shaft.

b) If the coefficient of friction between phosphor bronze and steel is about 0.34 dry or 0.17 greasy, what torque would you estimate could be transferred from shaft to disk with no slippage if the minimum diametral interference is a half of the maximum interference?

c) Approximately what hydraulic press capacity would you estimate might be needed to press the shaft out of the disk after it had been assembled?

For commercial bronze (C-52100): S_{ut}=55000 psi , S_y=30000 psi , E=$16\text{x}10^5$ psi , ν=0.35

For cold-drawn 1020 steel: S_{ut}=61000 psi , S_y=51000 psi , E=$30\text{x}10^6$ psi , ν=0.30



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Midterm Examination

MECH 441/2 Oct. 29, 2003

$\sigma_y = 20$
$\sigma_x = -80$

Problem 1 (33 marks)

$$\sigma_1, \sigma_3 = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{-80+20}{2} \pm \sqrt{\left(\frac{-80-20}{2}\right)^2 + 50^2}$$

$$\sigma_1 = -30 + 70.71 = 40.71 \text{ MPa}; \quad \sigma_2 = 0; \quad \sigma_3 = -30 - 70.71 = -100.71 \text{ MPa}$$

$$\tau_{12} = \frac{1}{2}\left|\sigma_1 - \sigma_2\right| = \frac{1}{2}\left|40.71 - 0\right| = 20.355 \text{ MPa}$$

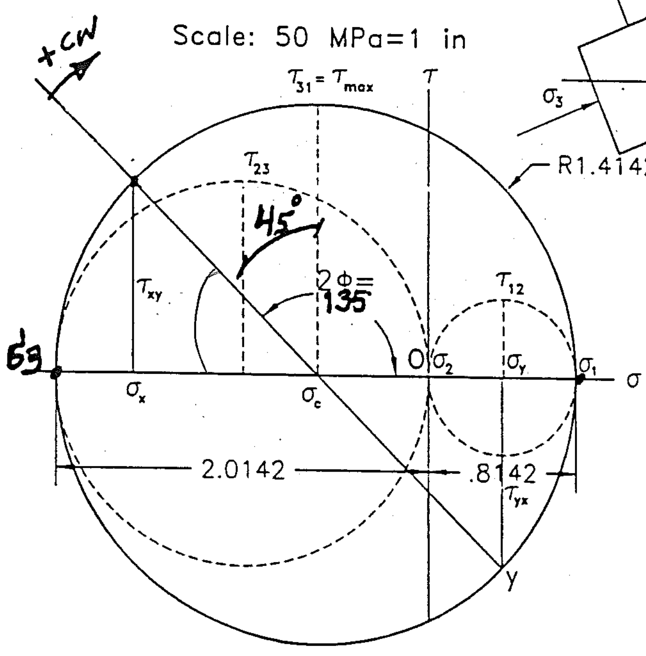
$$\tau_{23} = \frac{1}{2}\left|\sigma_2 - \sigma_3\right| = \frac{1}{2}\left|0 - (-100.71)\right| = 50.355 \text{ MPa}$$

$$\tau_{31} = \frac{1}{2}\left|\sigma_3 - \sigma_1\right| = \frac{1}{2}\left|-100.71 - 40.71\right| = 70.71 \text{ MPa} = \tau_{max}$$

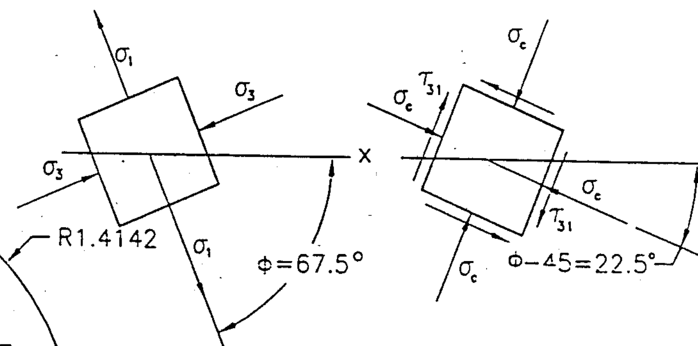
$$2\phi = \tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = \tan^{-1}\left[\frac{2(50)}{-80-20}\right] = \tan^{-1}\left(\frac{100}{-100}\right) = -45 + 180 = 135°$$

$$\phi = 67.5°$$

ANS



Mohr's Circles



Maximum shear stress theory (Tresca):

$$N_s = \frac{S_y}{2\tau_{max}} = \frac{320}{2(70.71)} = 2.26 \qquad \text{ANS}$$

Maximum distortion energy theory (von Mises):

$$\sigma' = \sqrt{\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2}$$

$$= \sqrt{40.71^2 - 40.71(-100.71) + (-100.71)^2}$$

$$\sigma' = 126.094 \text{ MPa}$$

$$N_s = \frac{S_y}{\sigma'} = \frac{320}{126.094} = 2.538 \qquad \text{ANS}$$

Maximum normal stress theory:

$$N_s = \frac{S_y}{|\sigma_3|} = \frac{320}{100.71} = 3.177 \qquad \text{ANS}$$

$$\theta = \frac{F}{2EI}\left[\left(1-\frac{\cancel{L}}{2\cancel{L}}\right)\left(\frac{\ell}{2}\right)^2 - \left(\frac{L}{2}-\frac{L}{2}\right)^2\right.$$

$$\left. + \frac{\cancel{L}}{2\cdot 3\cdot \cancel{L}}\left(-\left(\frac{L}{2}\right)^2 + 3\frac{L}{2}\cdot L - 2\ell^2\right)\right.$$

$$\theta = \frac{F}{2EI}\left[\left(\frac{1}{2}\right)\left(\frac{\ell^2}{4}\right) - 0 + \frac{1}{6}\left(\frac{-L^2}{4} + \frac{3}{2}L^2 - 2L^2\right)\right]$$

$$= \frac{F}{2EI}\left[\frac{L^2}{8} + \frac{1}{6}\left[\frac{-L^2+6L^2-8L^2}{4}\right]\right.$$

$$= \frac{F}{2EI}\left[\frac{L^2}{8} - \frac{3L^2}{24}\right]$$

$$= \frac{F}{2EI}\left[\frac{3L^2 - 3L^2}{24}\right]$$

$=$

Problem 2 (33 marks)

Bending stress due to the impact moment:

$$\sigma_i = \frac{6M_i}{wt^2} = \frac{6M_i}{3(1)^2} = 2M_i = S_y = 40000 \text{ psi}$$

$$M_i = \frac{\sigma_i}{2} = \frac{40000}{2} = 20000 \text{ lbfin}$$

Static bending moment $M_{st} = R_1 \frac{\ell}{2} = \frac{W}{2}\frac{\ell}{2} = \frac{W\ell}{4}$

$$M_{st} = \frac{78(60)}{4} = 1170 \text{ lbfin}$$

Impact factor $\frac{F_i}{W} = \frac{M_i}{M} = \frac{20000}{1170} = 17.094$

But

$$\frac{F_i}{W} = 1 + \sqrt{1 + \frac{2\eta h}{\delta_{st}}}$$

$\eta = 1.0$ the mass of the beam neglected

$\delta_{st} = \frac{F\ell^3}{48IE}$ for simply supported beam

$$\boxed{\delta_{st} = \frac{W\ell^3}{48IE} = \frac{78(60)^3 12}{48(3)1^3(30)10^6} = 0.0468 \text{ in}}$$

$$1 + \sqrt{1 + \frac{2h}{\delta_{st}}} = 17.094$$

$$h = \frac{\delta_{st}}{2}(16.094^2 - 1) = \frac{0.0468}{2}(16.094^2 - 1) = 6.038 \text{ in} \quad \underline{\text{ANS}}$$

Problem 3 (34 marks)

(a) Pressure at contact surface

Disk inner radius:

$$\sigma_r = -p\,; \quad \sigma_t = \frac{1+\psi_e^2}{1-\psi_e^2}p = \frac{1+(3/10)^2}{1-(3/10)^2}p = \frac{109}{91.0}p = 1.1978p$$

von Mises stress:

$$\sigma' = \sqrt{\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2} = \sqrt{(1.1978p)^2 - 1.1978p(-p) + (-p)^2} = 1.90592p$$

σ' should not excced one half of the yield strength

$$\sigma' \leq \frac{1}{2} S_y = \frac{1}{2}(30000) = 15000 \text{ psi}$$

The maximum pressure corresponds to maximum stress

$$1.90592\, p_{max} = \sigma' = 15000 \text{ psi} \; ; \; p_{max} = \frac{15000}{1.90592} = 7870.21 \text{ psi}$$

Since stress at outher radius of the disc is $\sigma_t = \frac{2\psi_e^2}{1-\psi_e^2} p$; $\sigma_r = 0$

$\sigma' = \sigma_t = \frac{2(3/10)^2}{1-(3/10)^2} p = 0.1978\, p$; Therefore it is much lower,

The maximum interference causes the maximum pressure

$$\delta_{max} = \frac{p_{max} d}{2E_e}\left(\frac{1+\psi_e^2}{1-\psi_e^2} + \nu_e\right) + \frac{p_{max} d}{2E_i}\left(\frac{1+\psi_i^2}{1-\psi_i^2} - \nu_i\right)$$

$$= \frac{7870.21(3)}{2(16)10^5}\left(\frac{1+(3/10)^2}{1-(3/10)} + 0.35\right) + \frac{7870.21(3)}{2(30)10^6}\left(\frac{1+(2/3)^2}{1-(2/3)^2} - 0.30\right)$$

$$= 0.01142 + 0.00059 = 0.01201 = 0.0120 \text{ in}$$

Diametral interference:

$$\Delta_{max} = 2\delta_{max} = 2(0.0120) = 0.024 \text{ in}$$ <u>ANS</u>

(b) $\Delta_{min} = \frac{1}{2}\Delta_{max} = \frac{1}{2}(0.024) = 0.012 \text{ in}$

$$p_{min} = \frac{\Delta_{min}}{\Delta_{max}} p_{max} = \frac{1}{2} p_{max} = \frac{1}{2}(7870.21) = 3935.1 \text{ psi}$$

Torque load rating:

$$T = \frac{1}{2}\pi d^2 l\, p_{min}\, \mu_{min} = \frac{1}{2}\pi(3)^2 4(3935.1)\,0.17$$

$$T = 37{,}829.2 \text{ lbfin}$$ <u>ANS</u>

(c) Press force:

$$F_a = \pi d l\, p_{max}\, \mu_{max} = \pi(3)4(7870.21)\,0.34$$

$$F_a = 100{,}878.0 \text{ lbf}$$ <u>ANS</u>