

**CONCORDIA UNIVERSITY**  
**DEPARTMENT OF MECHANICAL ENGINEERING**  
**Instrumentation and Measurement, ENGR 373/2 X**

**MIDTERM EXAMINATION**  
**October 18, 2000**

**Instructor:** Chun-Yi Su

**Time:** 1.15 hours

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**Special instructions:**

1. No books or notes are permitted
  2. Answer all questions
  3. Explain your solutions and show intermediate calculations
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**NAME OF STUDENT:** \_\_\_\_\_

**I.D.:** \_\_\_\_\_

NAME OF STUDENT: \_\_\_\_\_

I.D.: \_\_\_\_\_

**Problem 1** (8 points)

The specifications of an electro-mechanical pressure measurement system are listed in the manufacturer's catalog, as follows:

Range:	±500 psi
Output at 100 psi:	1 V at 20°C
Output at 500 psi:	5 V at 20°C
Linearity	±0.5% of the range
Hysteresis:	±0.2% of the range
Accuracy of the measurement system:	±0.5% of the reading
Thermal zero drift:	0.01% of F.S.O./°C
Sensitivity drift:	0.01%/°C

Determine the following:

- Static sensitivity of the measurement system.
- Overall error based upon the instrument's range, if it is operated at 20°C.
- Change in output, when the test temperature increases to 30°C under an input pressure of 300 psi.

Static Sensitivity = ? 2       $V_H = \sqrt{V_c + V_o}$

a)  $y_L = \pm \frac{0.5}{100} \cdot 500 \text{ psi} = \pm 2.5 \text{ psi}$

$y_H = \pm \frac{0.2}{100} \cdot 500 \text{ psi} = \pm 1 \text{ psi}$

$y_Z = \frac{0.01}{100\%} \cdot 100 \text{ psi} \cdot 20^\circ\text{C} = \pm 0.2 \text{ psi}$

$y_S = \frac{0.01}{100\%} \times 20 \times 100 \text{ psi} = \pm 0.2 \text{ psi}$

$y_T = y_L + y_Z + y_H + y_S = \pm 3.9 \text{ psi}$

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b)  $e_o = \sqrt{e_L^2 + e_Z^2 + e_H^2 + e_S^2}$   
 $= \sqrt{2.5^2 + 1^2 + 0.2^2 + 0.2^2}$

$e_o = \pm 2.711$

How you get this, don't you study the chapter 1?

c)  $y = KA + K_1 K_2 M(\omega) M(\omega_2) \sin(\omega t + Q_1 + Q_2)$

$$a) \quad K = \frac{5-1}{500-100} = \frac{4V}{400\text{psi}} = 0.01 \text{ V/psi}$$

$$b) \quad \text{Output range} = \pm 5V \Rightarrow FSO = y_{\max} - y_{\min} = 10V$$

$$e_i = FSO \frac{0.5}{100} = 0.05V$$

$$e_H = FSO \frac{0.2}{100} = 0.02V$$

$$e_A = FSO \frac{0.5}{100} = 0.05V$$

$$e_T = \frac{0.01}{100} \cdot FSO \cdot \Delta T = 0.1V$$

$$e_s = \frac{0.01}{100} \cdot FSO \cdot \Delta T = 0.1V$$

$$e_T = \frac{0.01}{100} \cdot FSO \cdot 30 = 0.03V$$

$$e_s = \frac{0.01}{100} \cdot FSO \cdot 20 = 0.02V$$

$$e = \sqrt{e_i^2 + e_H^2 + e_A^2 + e_T^2 + e_s^2}$$

$$c) \quad e_T^{\Delta T} = \frac{0.01}{100} \cdot FSO \cdot \Delta T = \frac{0.01}{100} \cdot 10 \cdot 10 = 0.01V$$

$$e_s^{\Delta T} = \frac{0.01}{100} \cdot \text{reading} \cdot \Delta T = \frac{0.01}{100} \cdot 300 \cdot 0.01 \cdot 10 = 0.003V$$

$$\Delta y = 0.01 \times 0.003 = 0.013V$$

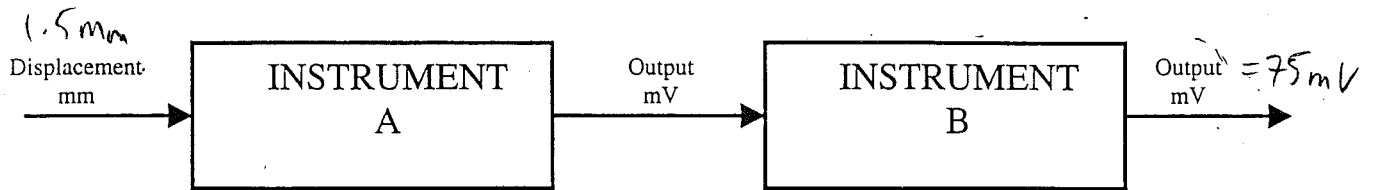
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**Problem 2** (4 points)

Two instruments with linear input-output characteristics are connected as shown. The static sensitivity of instrument B is 2.5 V/V. For a displacement input of 1.5 mm, the measurement system yields an output of 75 mV. Compute the static sensitivity of instrument A:

- a) If there is no zero drift.
- b) If the zero drift of instrument A is 2.5 mV and the zero drift of instrument B is -1.0 mV.



a)  $V_{oB} = mh + c$  ← Static sensitivity, zero drift

$75 \text{ mV} = 2.5 \frac{\text{V}}{\text{V}} h$

$h_B = 0.03$

$1.5 \text{ mm} \pm 0.03$

$\frac{2.5}{4}$

$V_{oB} = mh + c$

$75 \text{ mV} = 2.5 \frac{\text{V}}{\text{V}} h - 1.0 \text{ mV}$

$h_B = 0.0304$

$1.5 \text{ mm} \pm 0.0304$

a)  $k_1, k_2 = \text{input} = 0.075$   
 $k_1 = 2.5, k_2 = 1.5 = 0.075$   
 $k_1 =$

a)  $FSD = 1.5 \text{ mm}$   
 $\text{output} = 75 \text{ mV}$

$$y_0 = 75 \text{ mV} = 2.5 \frac{\text{V}}{\text{V}} \times y_A$$

$$y_A = 0.03 \text{ V}$$

$$y_A = K_A \cdot FSD$$

$$0.03 \text{ V} = K_A \cdot 1.5 \text{ mm}$$

$$K_A = 0.02 \text{ V/mm}$$

b)  $y_{2A} = 2.5 \text{ mV}, y_{2B} = -1.0 \text{ mV}$

$$y_B = 75 \text{ mV} = \left( 2.5 \frac{\text{V}}{\text{V}} \times y_A \right) + (-1.0 \text{ mV})$$

$$76 \text{ mV} = \left( 2.5 \frac{\text{V}}{\text{V}} \right) \times y_{2A}$$

$$y_A = 0.0304 \text{ V}$$

$$y_A = (FSD \times K_A) + y_{2A}$$

$$0.0304 \text{ V} = (1.5 \text{ mm} \cdot K_A) + 0.0025 \text{ V}$$

$$0.0279 \text{ V} = 1.5 \text{ mm} \cdot K_A$$

$$K_A = 0.0186 \text{ V/mm}$$

**Problem 3 (8 points)**

A seismic measurement system is used to measure the acceleration. The ringing frequency of the instrument is 954 Hz, and its damping ratio is specified as 0.3. The static sensitivity of the accelerometer is 50vm/m/s<sup>2</sup>.

- Determine the frequency range for which the amplitude dynamic error will be less than 5%.
- Recommend design modifications to minimize the amplitude dynamic error and phase error. Explain your answers briefly.

The above instrument is used in conjunction with a first order signal recorder and amplifier with time constant of 25 ms and amplification factor of 10 v/v. Determine the steady state output of the measure system, when subjected to an acceleration signal of the form:

$$\ddot{x}(t) = -9.81 \pm 2.5 \sin(1000\pi) \text{ m/s}^2$$

$$\tau = 25 \text{ ms}$$

$$A = 10 \text{ v/v}$$

a)  $f_d = 954 \text{ Hz}$ ,  $\zeta = 0.3$ ,  $KA = 50 \text{ vm/m/s}^2$   
 sinusoidal (ringing)

$$M(\omega) = \frac{1}{\left[ \left( 1 - \frac{\omega^2}{\omega_n^2} \right)^2 + \left( 2\zeta \frac{\omega}{\omega_n} \right)^2 \right]^{1/2}}$$

$$= \frac{1}{\left[ \left( 1 - \frac{\omega^2}{(6283.59)^2} \right)^2 + \left( 2(0.3) \frac{\omega}{6283.59} \right)^2 \right]^{1/2}}$$

$$\omega_d = 2\pi f_d$$

$$= 2\pi (954 \text{ Hz})$$

$$= 5994.16 \text{ rad/s}$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

$$\omega_n = \frac{5994.16 \text{ rad/s}}{\sqrt{1 - 0.3^2}}$$

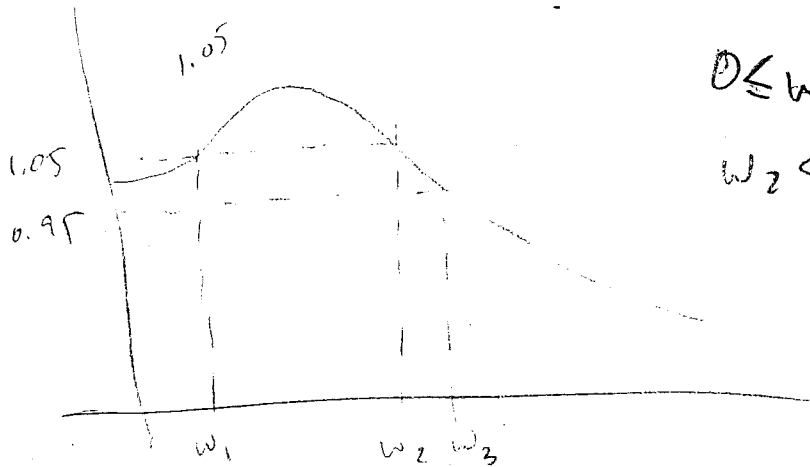
$$= 6283.59 \text{ rad/s}$$

$$\omega_r = \omega_n \sqrt{1 - \zeta^2}$$

$$\frac{2}{8}$$

$$f_n = f_d \sqrt{1 - \zeta^2}, \quad 0.95 \leq M(\omega) \leq 1.05$$

Change DAMPING RATIO OF 0.707.



$$0 \leq \omega < \omega_1$$

$$\omega_2 < \omega < \omega_3$$

$$f_1 = 954$$

$$\omega_{st} = \omega_n \sqrt{1 - \zeta^2}$$

$$\omega_n = 6283.587 \text{ rad/s}$$

$$|m - 1| \leq 0.05$$

$$0.95 \leq m \leq 1.05$$

$$m(\omega) = \frac{\omega^2 / \omega_n^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}}$$

$$\phi = \tan^{-1} \frac{2\zeta \frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$$y(t) = A_1 K_1 K_2 + A_2 K_1 K_2 M_1 M_2 \cdot \sin(\omega t + \phi_1 + \phi_2)$$

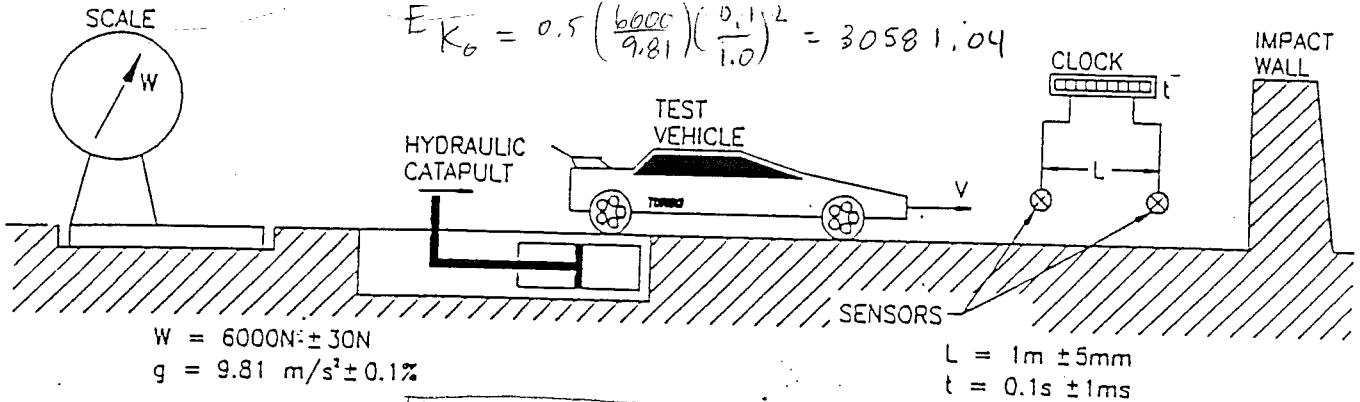
$$M_1 = 0.01273, \quad \phi_1 = -89.27^\circ$$

$$M_2 = 1.23797, \quad \phi_2 = -21.8^\circ$$

**Problem 4 (5 points)**

Crash tests are performed to assess the crashworthiness of automobiles in terms of kinetic energy of the vehicle ( $0.5mv^2$ ) just before the impact. The mass of the vehicle is estimated from the measured weight of the vehicle, such that  $m=W/g$ . The final vehicle speed is estimated by clocking the time required to travel between the two instrumented markers, as shown in the figure, such that  $v=L/t$ . Compute the absolute uncertainty in the kinetic energy measurement.

$W=6000\text{ N}\pm 30\text{N}; L=1.0\text{m}\pm 5\text{mm}; g=9.81\text{ m/s}^2\pm 0.1\%; t=0.1\text{s}\pm 1\text{ms}.$



$E_{K_0} = 0.5mv^2$

$i$	$x_i$	$R_i^+$	$R_i^-$	$\delta R_i^+$	$\delta R_i^-$	$\delta R_i$
1	m	30429.65	30735.49	-151.39	154.45	152.92
2	v	30279.01	30890.72	-302.03	309.68	305.86

$R_{i_1}^+ = f\left(\frac{6000+30}{9.81+0.0981}, \frac{1.0}{0.1}\right) = 30429.65$

$R_{i_1}^- = f\left(\frac{6000-30}{9.81-0.0981}, \frac{1.0}{0.1}\right) = 30735.49$

$\delta R_{i_1}^+ = R_{i_1}^+ - R_0 = 30429.65 - 30581.04 = -151.39$

$\delta R_{i_1}^- = R_{i_1}^- - R_0 = 30735.49 - 30581.04 = 154.45$

$\delta R_{i_1} = \frac{|\delta R_{i_1}^+| + |\delta R_{i_1}^-|}{2} = 152.92$

2)  $R_{i_2}^+ = f\left(\frac{6000}{9.81}, \frac{1.0+0.005}{0.1+0.001}\right) = 30279.01$

$R_{i_2}^- = f\left(\frac{6000}{9.81}, \frac{1.0-0.005}{0.1-0.001}\right) = 30890.72$

$\delta R_{i_2}^+ = 30279.01 - 30581.04 = -302.03$

$\delta R_{i_2}^- = 30890.72 - 30581.04 = 309.68$

$\delta R_{i_2} = \frac{|\delta R_{i_2}^+| + |\delta R_{i_2}^-|}{2} = 305.86$

$U_k = \sqrt{\delta R_{i_1}^2 + \delta R_{i_2}^2} = \sqrt{(152.92)^2 + (305.86)^2}$

$U_k = \pm 341.95\text{ J/s}$

or  $30890.72 \pm 341.95\text{ J/s}$



CONCORDIA UNIVERSITY  
DEPARTMENT OF MECHANICAL ENGINEERING

MECH 373/2 : INSTRUMENTATION AND MEASUREMENT

**FINAL EXAMINATION**

Instructors: A. Kaushal and S. Rakheja

Date: 16 December 1996

Time: 09:30 - 12:30

Materials Allowed: None

Maximum Marks: 60

NOTES: All questions carry equal marks.  
Attempt all questions.

QUESTION # 1: Select only one appropriate answer for the following:

- i) Linearity of an instrument defines
- a) maximum deviation of the output from true value.
  - b) minimum change in input required to reflect a change in the output.
  - c) maximum deviation of the output from the 'best-fit' straight line.
  - d) none of the above.
- ii) The  $\chi^2$ -square test is applied to
- a) test the linearity of the measured data.
  - b) determine whether certain data points can be rejected.
  - c) test whether the data follows the Gaussian distribution.
  - d) none of the above.
- iii) Precision of an instrument expresses
- a) closeness of the data to the mean value. (precision)
  - b) closeness of the data to the true value. (accuracy)
  - c) closeness of the data to the median value.
  - d) none of the above.
- iv) A force transducer is used to measure a true force of 200 N. The measurements repeated 5 times resulted in the following readings.

Reading #	1	2	3	4	5
Force, N	197	201	200	198	199

The bias of the instrument is

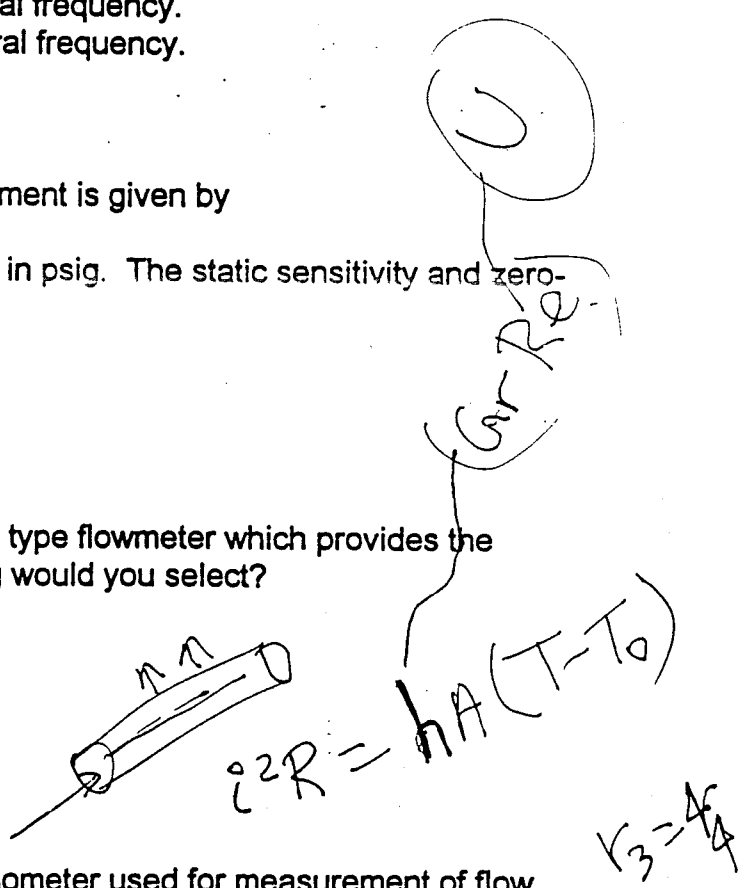
- a) 199.0 N; b) 1.0 N; c) 0.5 N; d) none of the a, b and c.
- v) The accuracy of the above instrument (based upon the readings) is
- a) 1.5%; b) 2.0%; c) 0.5%; d) none of the a, b and c.

a) 1.5%

Bias = diff  
between  
mean &  
true  
values

- vi) A measurement system comprises an instrument, a signal processing unit, a signal filter and a signal recorder. The uncertainties of the four units are  $\pm 1.5\%$ ,  $\pm 2.5\%$ ,  $\pm 0.5\%$  and  $\pm 2\%$ , respectively. The overall uncertainty of the measurements is approximately
  - a)  $\pm 2.1\%$
  - b)  $\pm 6.3\%$
  - c)  $\pm 3.6\%$
  - d) none of the above.
  
- vii) A  $350\Omega$  strain gage with gage factor of 2.2 is subjected to a longitudinal strain of  $160 \mu\text{m/m}$ ; the resulting change in resistance is approximately
  - a)  $123\text{m}\Omega$
  - b)  $25 \text{ m}\Omega$
  - c)  $46 \text{ m}\Omega$
  - d) none of the above.

$GF = \frac{\Delta R/R}{\Delta L/L}$
  
- viii) A seismic type accelerometer provides accurate dynamic measurements
  - a) at frequencies well below its natural frequency.
  - b) at frequencies well above its natural frequency.
  - c) at any frequency.
  - d) none of the above.
  
- ix) The output-input relationship of an instrument is given by
 
$$q_o = 0.1 + 2q_i$$
 where  $q_o$  is output in volts and  $q_i$  is input in psig. The static sensitivity and zero-drift of the instrument are
  - a)  $2 \text{ V/psig}$ ,  $50 \text{ mV}$
  - b)  $2 \text{ V/psig}$ ,  $100 \text{ mV}$
  - c)  $0.1 \text{ V/psi}$ ,  $2 \text{ V}$
  - d) none of the above.
  
- x) You are required to select an obstruction type flowmeter which provides the highest accuracy. Which of the following would you select?
  - a) orifice plate flowmeter.
  - b) nozzle flowmeter
  - c) venturi flowmeter
  - d) any of the above.

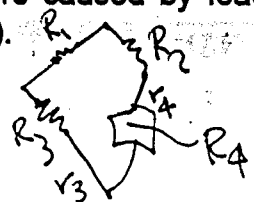


**QUESTION # 2:**

- a) Describe the principle of a hot wire anemometer used for measurement of flow rate.
- b) Discuss a method to minimize the errors caused by lead wire resistance in a Resistance Temperature Detector (RTD).

~~IF~~

$$\frac{R_1}{R_2} = \frac{R_4 + r_4}{R_3 + r_3}$$



$$R_3 + r_3 = R_4 + r_4$$

$$R_4 = R_3 + r_3 - r_4$$

- c) A venturi flowmeter is used to measure fluid flow rate in 0.1m diameter pipe. The difference in pressure ( $p_1$ ) measured at the pipe prior to the throat and the pressure ( $p_2$ ) measured at the throat is 20 kPa. Given that throat diameter is 6 cm and the coefficient of discharge  $C=0.98$ , determine the ideal and actual flow rates. Assume  $\rho = 800 \text{ kg/m}^3$ .

$$\text{Pressure drop across an obstruction: } \Delta P = \frac{\rho u_2^2}{2} \left[ 1 - \left( \frac{A_2}{A_1} \right)^2 \right]$$

### QUESTION # 3:

- a) The calibration of a 100 N strain-gage based force sensor at normal operating temperature of 22°C resulted in the calibration curve (I) shown below. The calibration curve obtained at a temperature of 8°C is indicated as curve II. Determine
- static sensitivity of the instrument at 22°C.
  - thermal sensitivity drift in %/°C.
  - zero-drift in %/°C.
- b) A 10 kΩ potentiometer is used to measure the displacement of a component and a voltmeter with internal resistance of 100 kΩ is used to measure the sensor output. Determine the loading error when the potentiometer slider is at 50% position and an excitation of 20 V is used.

### QUESTION # 4:

The gas pressure in a process is measured using a diaphragm type pressure sensor. The volume of gas contained in the sensor cavity is  $V=9.83 \text{ cc}$ , and the sensor is connected to the main gas pipe using a 4.5 cm long capillary tube of 0.8 mm diameter. The natural frequency of the capillary-cavity system is 50 Hz. Determine the percent attenuation of the gas pressure signal, when frequency of the pressure signal is 50 Hz.

Properties of gas at 25°C:  $\rho = 118 \text{ kg/m}^3$ ;  $\mu = 198 \cdot 10^{-5} \text{ kg/m}\cdot\text{s}$

Velocity of sound in gas:  $c = 20.04\sqrt{T}$ , where T is in °K

Damping ratio:  $\zeta = \frac{2\mu}{\rho c r^3} \sqrt{\frac{3LV}{\pi}}$ ; and  $\omega_n = \sqrt{\frac{3\pi r^2 c^2}{4LV}}$

**QUESTION # 5:**

A seismic instrument is designed to measure the acceleration through measurement of relative deflection of the seismic mass. A miniature LVDT is used to measure the deflection of the seismic mass, as shown. The undamped natural frequency of the seismic instrument is 10 kHz and the effective stiffness  $K=2\text{MN/m}$ .

- a) Determine the mass  $m$  and damping coefficient  $c$  required, if a damping ratio of 0.707 is desired.
- b) Determine the sensitivity of the accelerometer in volts/g at a frequency of 1500 Hz, given that the sensitivity of the LVDT is 250 mV/mm.

Handwritten notes and equations:

- $\delta E_0 = \frac{E_1 L}{2 + EA}$
- $E_1 = 1200$
- $y = 2\text{mm}$
- $\frac{\delta E_0}{E_1} = \frac{5R/R}{4 + 25R/R}$
- $\frac{3}{12}$  (circled)

**QUESTION # 6:**

A cantilever beam is designed to measure displacement  $y$  of a component attachment to its free end. It is known that the local strain at A is linearly related to deflection  $y$  of the free end in the  $\pm 10\text{mm}$  range. The local strain at A and B was measured to be  $1200 \mu\text{m/m}$  in C tension and compression, respectively, under  $2\text{mm}$  displacement of the free end.

It is desirable to measure the displacement using  $120\Omega$  strain gages with gage factor of 4. It is also required to enhance the sensitivity of instrument using two gages in a Wheatstone bridge configuration.

- a) Propose the layout of strain gages and Wheatstone bridge configuration using two strain gages, two fixed resistors and 6V DC power source.
- b) Determine the voltage output under  $y=2 \text{ mm}$ , and the voltage sensitivity of the measurement system.