Concordia
UNIVERSITY
Department of Mathematics & Statistics

48

| Course | Number | Section(s) |
|---|---|---|
| Mathematics | 205/4 | All |
| **Examination** | **Date** | **Pages** |
| Final | April 2003 | 3 |
| **Instructors** | | **Course Evaluator** |
| A. Boyarsky, P. Gauthier, R. Mearns, A. Padmanabhan, H. Proppe | | Y. Khidirov |

**Special Instructions**

- ▷ Calculators are **not** allowed.
- ▷ Tables of integrals are **not** allowed.

*MARKS*

[10] **1.** (a) Sketch the graph of the function

$$f(x) = \begin{cases} \sqrt{4-x^2}, & \text{if } -2 \le x \le 0 \\ 2-x, & \text{if } \ 0 < x \le 4 \\ -2, & \text{if } \ 4 < x \le 5 \end{cases}$$

Evaluate the definite integral $\int_{-2}^{5} f(x)\,dx$ by interpreting it in terms of area (do not antidifferentiate).

(b) Find the derivative of the function

$$F(x) = \int_0^{\cos x} \frac{\sqrt{1-t^2}}{t+2}\, e^{2t}\, dt$$

[15] **2.** Find the indefinite integrals:

(a) $\displaystyle\int \frac{x + \arctan x}{1 + x^2}\,dx$ (b) $\displaystyle\int (x^2+4)e^{2x}\,dx$ (c) $\displaystyle\int \frac{x^2 + x - 1}{x^2 + x}\,dx$

[15] **3.** Calculate the definite integrals:

(a) $\displaystyle\int_0^{\frac{\pi}{2}} \sin x \cos^4 x\,dx$ (b) $\displaystyle\int_0^2 x\sqrt{4 - x^2}\,dx$ (c) $\displaystyle\int_1^e x^2 \ln x\,dx$

[15] **4.** (a) Find the area bounded by the curves $y = 2x^2$, $x + y = 3$ and $x = 0$.

(b) Find the volume of the solid obtained by rotating the region bounded by the curve $y = 1 + \frac{1}{x}$, vertical lines $x = 1, x = 2$, and $x$-axis about the $x$-axis.

(c) Find the average value of the function $f(x) = e^{2x}$ on the interval $[0, 2]$.

[12] **5.** Evaluate the given improper integral or show that it diverges:

(a) $\displaystyle\int_4^{+\infty} \frac{1 + \sqrt{x}}{x\sqrt{x}}\,dx$ (b) $\displaystyle\int_0^8 \frac{dx}{\sqrt[3]{x}}$

[9] **6.** Find the limit of the sequence or show that it does not exist:

(a) $\displaystyle\left\{\frac{n^2 - 1}{n^2 + 1000}\right\}$ (b) $\displaystyle\left\{\frac{\sin(n^3)}{n^3}\right\}$ (c) $\{1 + (-1)^n\}$

[12] 7. Test each of the following series to determine if it is convergent or divergent:

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln^2 n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(n+1)\,5^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{5n+1}$

[12] 8. (a) Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{5^n}$

(b) Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^n}{n(n+1)}$

(c) Find the MacLaurin series for the function $f(x) = \dfrac{x}{1+x^2}$

[5] **Bonus Question**

Is the solution below correct? If not, what is wrong?

$$\int_{-1}^{1} \frac{dx}{x^2} = \int_{-1}^{1} x^{-2}\,dx = \left.\frac{x^{-1}}{-1}\right|_{-1}^{1} = \left.-\frac{1}{x}\right|_{-1}^{1} = -\left(\frac{1}{1} - \frac{1}{-1}\right) = -(1+1) = -2$$

**Concordia University**
**Department of Mathematics and Statistics**
**Final Examination**

| | |
|---|---|
| **Course:** | **MATH 205 Section BA** |
| **Date :** | **August 25th, 2003** |
| **Instructor :** | **Pierre Q. Gauthier** |
| **Special Instructions :** | **Calculators are NOT allowed** |

1. a) Sketch the graph of the function

$$f(x) = \begin{cases} -(x+2) & -4 \le x \le -2 \\ 2+\sqrt{4-x^2} & -2 \le x \le 0 \\ 2-\sqrt{4-x^2} & 0 \le x \le 2 \\ -(x-2) & 2 \le x \le 4 \end{cases}$$

Evaluate the integral $\int_{-4}^{4} f(x)dx$ by interpreting it in term of area.

b) Find the derivative of the function

$$F(x) = \int_{0}^{\tan x} \frac{\sqrt{1+t^2}}{t-1} dt$$

2. Find the indefinite integrals:

a) $\int \frac{x+\arcsin x}{\sqrt{1-x^2}} dx$ b) $\int (x^2-5)e^{7x} dx$ c) $\int \frac{(x+5)^2}{x^2+1} dx$

3. Calculate the definite integrals:

a) $\int_0^{\frac{\pi}{4}} \tan^8(x)\sec^2(x)dx$   b) $\int_1^e x^7 \ln(x)dx$   c) $\int_{16}^{25} \frac{\sqrt{x}}{\sqrt{x}+1}dx$

4. a) Find the area bounded by the curves $y = x^2 + 7$, $y = \frac{8}{x}$, $x = 0$ and $y = x^2$.

b) Find the volume of the solid obtained by rotating the region bounded by the curve $y = x^3\sqrt{8 - x^3}$, $0 \le x \le 2$, about the x-axis.

c) Find the average value of the function $f(x) = x\ln(x)$ on the interval [1,e].

5. Evaluate the given improper integral or show that it diverges:

a) $\int_9^{\infty} \frac{1+x^2}{x^3\sqrt{x}}dx$   b) $\int_0^{32} \frac{2}{\sqrt[5]{x}}dx$

6. Find the limit of the sequene or show that it does not exist:

a) $\left\{\frac{2n^7 + 4n^3 + 111}{n^7 - 15.2}\right\}$   b) $\left\{\frac{\cos(n^4)}{n^4}\right\}$   c) $\{\cos(n\pi) + (-1)^n\}$

7. Test each of the following series to determine if it converges or diverges:

a) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$   b) $\sum_{n=1}^{\infty} \frac{10^n(n+2)}{n!}$   c) $\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{1-n^3}$

(5)

8. a) Find the sum of the following series: $\sum_{n=1}^{\infty} \frac{(-7)^n}{8^n}$

b) Find the interval of converges of the power series: $\sum_{n=1}^{\infty} \frac{(x+5)^n}{n(n+2)}$

c) Find the MacLaurin series for the function: $f(x) = \arctan(x)$

**Bonus Question**

What is wrong with this proof that 0 = 1?

Integrating: $\int_0^1 \frac{1}{x} dx$ by parts, let $u = \frac{1}{x}$ and $dv = dx$, so $du = \frac{-1}{x^2} dx$ and $v = x$.

Therefore:

$$\int_0^1 \frac{1}{x} dx = \frac{1}{x} * x - \int_0^1 x * \frac{-1}{x^2} dx = 1 + \int_0^1 \frac{1}{x} dx \quad => \quad \int_0^1 \frac{1}{x} dx = 1 + \int_0^1 \frac{1}{x} dx \quad \text{or} \quad 0 = 1 \text{ !!}$$

Concordia UNIVERSITY

Department of Mathematics & Statistics

88

| Course | Number | Section(s) |
| --- | --- | --- |
| Mathematics | 205/1 | All |
| **Examination** | **Date** | **Pages** |
| Final | August 2002 | 4 |
| **Instructors** | | **Course Evaluator** |
| P. Olivares | | Y. Khidirov |

**Special Instructions**

- ▷ Calculators are allowed.
- ▷ Tables of integrals are not allowed.

*MARKS*

[12] **1.** (a) Give lower and upper estimates for the area bounded by the graph of the function $f(x) = \frac{2}{x} + 3$, vertical lines $x = 1$ and $x = 4$ and $x$-axis by using three approximating rectangles. Sketch the graph and the rectangles.

(b) Sketch the graph of the function

$$f(x) = \begin{cases} x+3, & \text{if } -3 \le x \le 0 \\ 1+\sqrt{4-x^2}, & \text{if } \quad 0 < x \le 2 \\ 1, & \text{if } \quad 2 < x \le 3 \end{cases}$$

Evaluate the definite integral $\int_{-3}^{3} f(x)\,dx$ by interpreting it in terms of area.

(c) Find the derivative of the function

$$F(x) = \int_1^{x^3} t\sqrt{1+\frac{1}{t}}\,dt$$

[16] **2.** Find the indefinite integrals:

(a) $\displaystyle\int \frac{\sin x}{3-\cos x}\,dx$ (b) $\displaystyle\int e^{\sqrt{x}}\,dx$

(c) $\displaystyle\int (1+\tan x)\sec^2 x\,dx$ (d) $\displaystyle\int \frac{x^2-5x+9}{x^2-5x+6}\,dx$

[12] **3.** Calculate the definite integrals:

(a) $\displaystyle\int_0^1 4x\sqrt{x^2+3}\,dx$ (b) $\displaystyle\int_1^2 x\ln x\,dx$ (c) $\displaystyle\int_0^1 \frac{(x+1)}{x^2+1}\,dx$

[10] **4.** (a) Find the area bounded by the curves $y=\cos x$, $y = x - \frac{\pi}{2}$ and $x$-axis.

(b) Find the volume of the solid obtained by rotating the region bounded by the curve $y = x - x^2$ $(0 \le x \le 1)$ about the $x$-axis.

[10] **5.** (a) A spring with a natural length of 10 feet exerts a force of 12 pounds when it is stretched to 12 feet. How much work against the spring does it take to stretch it from its natural length to a length of 14 feet?

(b) Find the average value of the function

$$f(x) = \sin 2x$$

on the interval $[0, \frac{\pi}{2}]$.

[8] **6.** Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

(a) $\displaystyle\int_0^1 \frac{dx}{\sqrt[4]{x}}$ (b) $\displaystyle\int_1^\infty \frac{dx}{\sqrt[4]{x}}$

[8] **7.** Given a sequence $a_n = \dfrac{n+1}{2n}$,

(a) Show that the sequence is decreasing.

(b) Show that the sequence is bounded.

(c) Find the limit of the sequence.

[12] **8.** Test each of the following series to determine if it is convergent or divergent:

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{(n+1)!}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{3n-7}$

[12] **9.** (a) Find the sum of the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{2^n}$

(b) Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{3^n}$$

**9.** (c) Find the MacLaurin series for the function

$$f(x) = x^2 e^{2x}$$

(Hint: Start from the MacLaurin series for $e^x$.)

[5] **Bonus Question**

Is the solution below correct? If not, what is wrong?

$$\int_{-1}^{1} \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_{-1}^{1} = \left(-\frac{1}{1}\right) - \left(-\frac{1}{-1}\right) = -1 - 1 = -2$$

# Concordia UNIVERSITY

**Department of Mathematics & Statistics**

| Course | Number | Section(s) |
|---|---|---|
| Mathematics | 205/4 | All |
| **Examination** | **Date** | **Pages** |
| Final | May 2002 | 4 |
| **Instructors** | | **Course Evaluator** |
| M. Bertola, P. Bracken, J. Brody, H. Proppe, B. Rhodes | | Y. Khidirov |

**Special Instructions**

- ▹ Calculators are allowed.
- ▹ Tables of integrals are not allowed.

*MARKS*

[12] **1.** (a) Give lower and upper estimates for the area bounded by the graph of the function $f(x) = \frac{(x-2)^3}{9}$, $x$-axis and vertical line $x = 5$ by using three approximating rectangles. Sketch the graph and the rectangles.

(b) Sketch the graph of the function

$$f(x) = \begin{cases} 1, & \text{if } -2 \le x \le -1 \\ 1 + \sqrt{1-x^2}, & \text{if } -1 < x \le 1 \\ 2 - x, & \text{if } 1 < x \le 4 \end{cases}$$

Evaluate the definite integral $\int_{-2}^{4} f(x)\,dx$ by interpreting it in terms of area.

(c) Find the derivative of the function

$$F(x) = \int_e^{e^x} \frac{t\sin^2 t}{t^2+1}\,dt$$

5

[16] **2.** Find the indefinite integrals:

(a) $\displaystyle\int \frac{\cos x}{5+\sin x}\,dx$  (b) $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

(c) $\displaystyle\int \tan^4 x \sec^2 x\,dx$  (d) $\displaystyle\int \frac{3x+11}{x^2-2x-3}\,dx$

[12] **3.** Calculate the definite integrals:

(a) $\displaystyle\int_0^1 \frac{x}{\sqrt{x^2+4}}\,dx$  (b) $\displaystyle\int_1^{e^2} x^2 \ln x\,dx$  (c) $\displaystyle\int_0^2 \frac{(x+1)^2}{x^2+1}\,dx$

[10] **4.** (a) Find the area between the curves $y = \sin x$ and $y = \frac{4}{\pi^2}x^2$ $(0 \le x \le \frac{\pi}{2})$.

(b) Find the volume of the solid obtained by rotating the region bounded by the curve $y = x\sqrt{1-x^2}$ $(0 \le x \le 1)$ about the $x$-axis.

[10] **5.** (a) If a force of 2000 N is required to extend a certain spring to 4 cm longer than its natural length, how much work is done to extend it that far?

(b) Find the average value of the function

$$f(x) = \frac{e^{\frac{1}{x}}}{x^2}$$

on the interval [1, 3].

6

[8] **6.** Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

(a) $\displaystyle\int_0^8 \frac{\sqrt[3]{x}}{x}\,dx$ (b) $\displaystyle\int_8^\infty \frac{\sqrt[3]{x}}{x}\,dx$

[8] **7.** Given a sequence $a_n = \dfrac{2}{n^2+1}$,

(a) Show that the sequence is decreasing.

(b) Show that the sequence is bounded.

(c) Find the limit of the sequence.

[12] **8.** Test each of the following series to determine if it is convergent or divergent:

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2\,2^n}{n!}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2n-5}$

[12] **9.** (a) Find the sum of the series $\displaystyle\sum_{n=0}^{\infty}(-5)^{-n}$

(b) Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{2^{2n}}$$

**9.** (c) Find the MacLaurin series for the function

$$f(x) = \frac{e^x - 1}{x}$$

(Hint: Start from the MacLaurin series for $e^x$.)

[5] **Bonus Question**

Is the solution below correct? If not, what is wrong?

$$\int_{-e}^{1} \frac{dx}{x} = \ln|x|\Big|_{-e}^{1} = \ln|1| - \ln|-e| = \ln 1 - \ln e = 0 - 1 = -1$$

8

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**Department of Mathematics & Statistics**

| Course | Number | Section(s) |
|---|---|---|
| Mathematics | 205/2 | All |
| **Examination** | **Date** | **Pages** |
| Final | December 2002 | 3 |
| **Instructors** | | **Course Evaluator** |
| A. Boyarsky, Y. Khidirov, B. Rhodes | | Y. Khidirov |

**Special Instructions**

- ▷ Calculators are **not** allowed.
- ▷ Tables of integrals are **not** allowed.

*MARKS*

[10] **1.** (a) Sketch the graph of the function

$$f(x) = \begin{cases} -x-1, & \text{if } -3 \le x \le 0 \\ -\sqrt{1-x^2}, & \text{if } 0 < x \le 1 \\ x-1, & \text{if } 1 < x \le 2 \end{cases}$$

Evaluate the definite integral $\int_{-3}^{2} f(x)\,dx$ by interpreting it in terms of area (do not antidifferentiate).

(b) Find the derivative of the function

$$F(x) = \int_1^{x^3} \sqrt{t^2+1}\, e^t\, dt$$



(9)

[15] **2.** Find the indefinite integrals:

(a) $\displaystyle\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx$  (b) $\displaystyle\int \frac{1}{x^2}\cos\left(\frac{1}{x}\right)dx$  (c) $\displaystyle\int \sin^2 x\,\cos^3 x\,dx$

[15] **3.** Calculate the definite integrals:

(a) $\displaystyle\int_0^1 \frac{2x\,dx}{\sqrt{16-7x^2}}$  (b) $\displaystyle\int_0^\pi (2x^2+4x+7)\cos 2x\,dx$  (c) $\displaystyle\int_1^2 \frac{dx}{x(1+x^2)}$

[15] **4.** (a) Find the area bounded by the curves $y=x^3$, $y=\dfrac{1}{x}$, $y=0$ and $x=2$.

(b) Find the volume of the solid obtained by rotating the region bounded by the curve $y=\cos x$ $\left(-\frac{\pi}{2}\le x\le\frac{\pi}{2}\right)$ about the $x$-axis.

(c) Find the average value of the function $f(x)=\ln x$ on the interval $[1,e]$.

[12] **5.** Evaluate the given improper integral or show that it diverges:

(a) $\displaystyle\int_1^\infty \frac{x^2+1}{x^4}\,dx$  (b) $\displaystyle\int_0^{\frac{\pi}{2}} \cot x\,dx$

[9] **6.** Given a sequence: $\{0, \frac{1}{4}, \frac{2}{6}, \frac{3}{8}, \frac{4}{10}, \ldots\}$

(a) Find the general term of the sequence.

(b) Show that the sequence is bounded.

(c) Find the limit of the sequence.

10

[12] **7.** Test each of the following series to determine if it is convergent or divergent:

(a) $\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n}$ (b) $\sum_{n=1}^{\infty} \frac{(n-1)\,3^n}{n!}$ (c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{4n-7}$

[12] **8.** (a) Find the sum of the series $\sum_{n=2}^{\infty} \frac{1}{3^n}$

(b) Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{3^n}$

(c) Find the MacLaurin series for the function $f(x) = x^2 e^{-x}$.
(Hint: Start from the MacLaurin series for $e^x$.)

[5] **Bonus Question**

Is the solution below correct? If not, what is wrong?

$$\int_0^3 \frac{dx}{x-2} = \ln|x-2|\Big|_0^3 = \ln|3-2| - \ln|0-2| = \ln 1 - \ln 2 = -\ln 2$$

11