ENGR370 MODELING AND ANALYSIS OF LINEAR PHYSICAL SYSTEMS – Test No.1 – Winter 2000

Time: One hour

1. Fig.1 shows the graph of a network.
   (a) List all the trees of the network.
   (b) Taking 24 as the tree and node(c) as the reference, write the matrices [A] and [B].
   (c) Verify $[A].[B]^T = 0$. (2 + 3 + 2 = 7 marks)

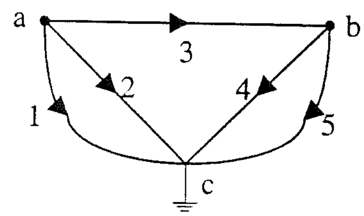

Fig.1

2. The following experimental data have been obtained in respect of two elements. Identify each element and obtain its magnitude and units. Give reasons for your answers. (3 + 3 = 6 marks)

Element A

Torque N.cm	Angular velocity (radians/second)
50	1
100	2
150	3

Element B

Force, N	Time, seconds	Velocity, m/s
8	0	0
8	1	3
8	2	6
8	3	9

3. Fig.3 shows an electrical network. Obtain the analogous mechanical (translational) circuit. Also, write the circuit giving the impedances and other elements in the Laplace transform domain.(Give reasons for your answers). (4 + 3 = 7 marks)

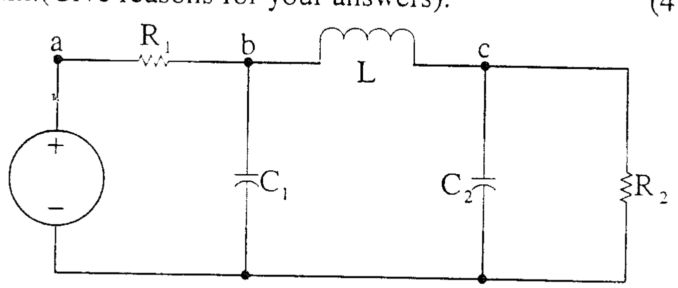

Fig.3.

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ENGR370 – MODELING AND ANALYSIS OF LINEAR PHYSICAL SYSTEMS
Test No.2 – Winter 2000

Time: One hour

1. Fig.1 shows the graph of a system. Taking the tree to be the branches 1,3,4, obtain the fundamental cut-set matrix.

(4 marks)

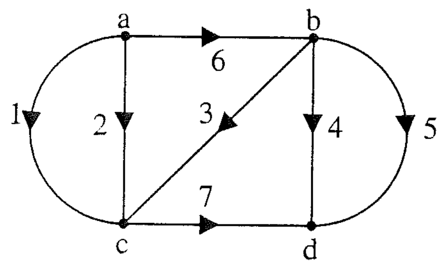

Fig.1.

2. Fig.2 shows a T-network. Starting from the definitions, obtain its y-parameters.

(10 marks)

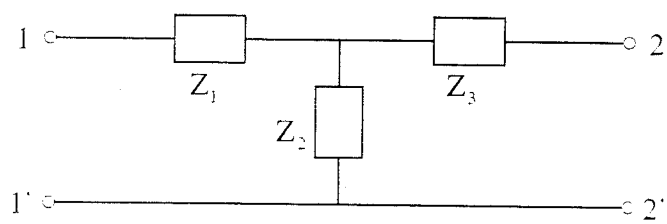

Fig.2.

3. The z-parameters of a network obtained after experimental measurements are given below:

$$[z] = \begin{bmatrix} \dfrac{2s+1}{s} & \dfrac{1}{s} \\ \dfrac{1}{s} & \dfrac{4s^2+1}{s} \end{bmatrix}$$

Obtain the equivalent T-network. Identify the various components and give their values.

(6 marks)

Engr 370

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Concordia UNIVERSITY

Course: Modeling and Analysis of Linear Physical Systems		Number: ENGR 370	Section: All Sections
Examination: Final	Date: April 2000	Time: 3 Hours	# of pages: 5
Instructor(s): Drs. A.K.W. Ahmed and V. Ramachandran			

Materials allowed: [X] No [ ] Yes (Please specify)

Calculators allowed: [ ] No [X] Yes

Students are allowed the use ONLY of non-programmable calculators WITHOUT text display.

Special Instructions:

Attempt all questions. Please number and begin each question on a new page.
Show all steps clearly in neat and legible handwriting (preferably in ink).
Students are required to return question paper together with exam booklet(s).

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1. Fig.1 shows an R-C network, excited by a current source. The various component values are: $R_1 = R_2 = R_3 = 1\ \Omega$, $C = 0.5$ F and $I_s = 1$ A.

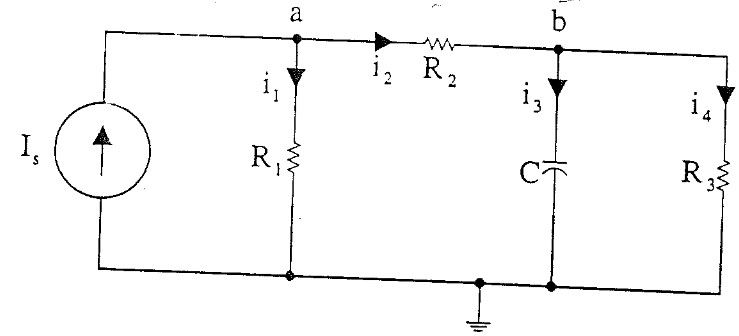


Fig.1.

(a) Taking R_1R_2 to be the tree, write the corresponding [A] matrix.
(b) Obtain the node-admittance matrix $[Y_n] = [A].[Y_B].[A]^T$.
(c) Hence, obtain $v_b(t)$. {Assume $v_b(0) = 0$}.
[Note: $L\{e^{at}\} = \frac{1}{s-a}$]

(12 marks)

2. (a) Starting from the definition, obtain the ABCD-parameters of (i) the series impedance Z_s, shown in Fig.2(a), and (ii) the shunt admittance Y_p, shown in Fig.2(b).

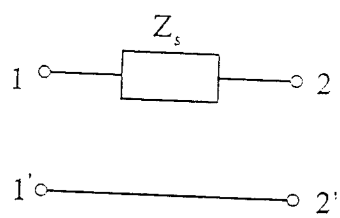


Fig. 2(a)

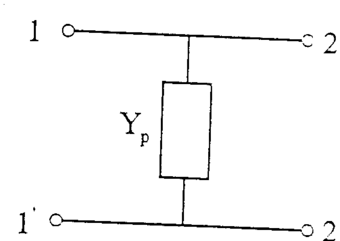


Fig. 2(b)

(6 marks)

(b) Write the electrical equivalent circuit of the mechanical translational system shown in Fig.2(c).

(3 marks)

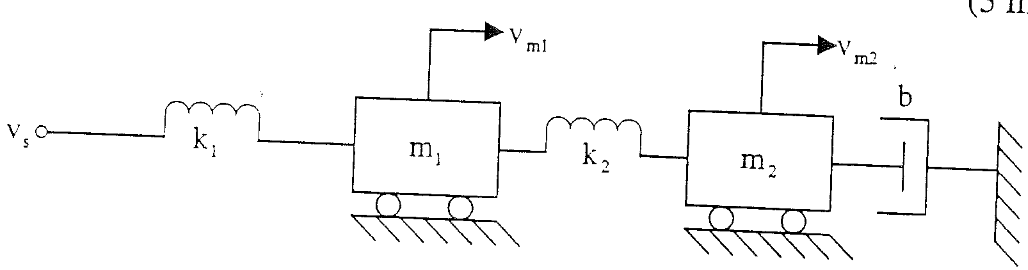


Fig.2(c)

(continued on page 3)

(c) Using the results of Figs.2(a) and (b), obtain the ABCD-parameters of the circuit shown in Fig.2(c). {v_s is the input and v_{m2} is the output)
(4 marks)

3. For the network shown in Fig.3,
(a) determine the y-parameters, starting from the definition.
(b) Hence, obtain the indefinite admittance matrix.

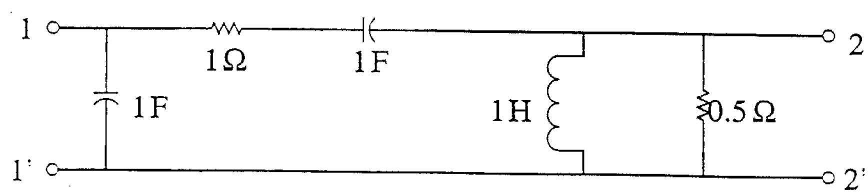


Fig.3.

(8 marks)

4. Fig.4 shows a fluid system.

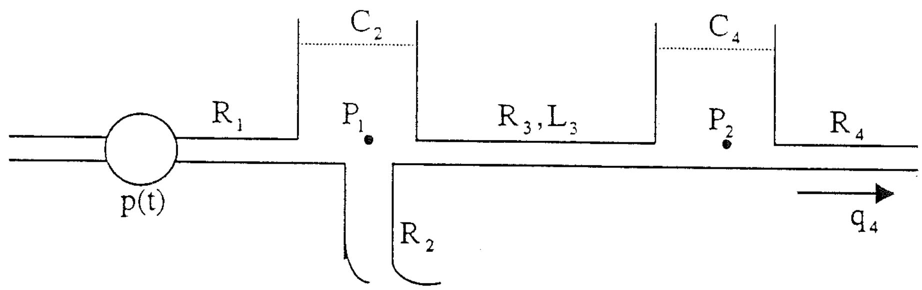


Fig.4

(a) Obtain the analogous electrical equivalent circuit in the Laplace-transform domain.
(b) Write the nodal equations.
(c) Write a signal-flow graph representing these equations.
(d) Hence, obtain $\frac{Q_4(s)}{P(s)}$.

(12 marks)

(continued on page 4)

5. A ladder network is shown in Fig.5.

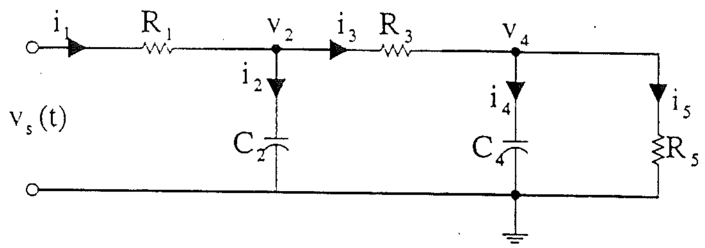


Fig.5.

(a) Obtain the state-equations in the matrix form.
(b) Hence, determine its characteristic equation.
(c) If $C_2 = C_4 = C$ and $R_1 = R_3 = R_5 = R$, show that the characteristic roots are simple and lie on the negative real axis.

(10 marks)

6. (a) Determine the order of the Butterworth low-pass filter which meets the following specifications:
   (i) The attenuation at 1000 Hz is 3 dB.
   (ii) The attenuation at 1500 Hz is greater than 12 dB.
   (b) Obtain the elemental values of a realization, when the terminating resistance is 1000 Ω.

(10 marks)

Table

Coefficients of the various Butterworth Polynomials upto order n = 8.

$$D_B(S) = a_n S^n + a_{n-1} S^{n-1} + \ldots + a_{n-k} S^{n-k} + \ldots + a_2 S^2 + a_1 S + a_0$$

Also, $a_{n-i} = a_i$

Order	
n=1	$a_1 = a_0 = 1$
n=2	$a_2 = a_0 = 1$ $a_1 = 1.414214$
n=3	$a_3 = a_0 = 1$ $a_2 = a_1 = 2$
n=4	$a_4 = a_0 = 1$ $a_3 = a_1 = 2.613216$ $a_2 = 3.414214$
n=5	$a_5 = a_0 = 1$ $a_4 = a_1 = 3.236068$ $a_3 = a_2 = 5.236068$
n=6	$a_6 = a_0 = 1$ $a_5 = a_1 = 3.863703$ $a_4 = a_2 = 7.464102$ $a_3 = 9.141620$
n=7	$a_7 = a_0 = 1$ $a_6 = a_1 = 4.493959$ $a_5 = a_2 = 10.097835$ $a_4 = a_3 = 14.591794$
n=8	$a_8 = a_0 = 1$ $a_7 = a_1 = 5.125831$ $a_6 = a_3 = 21.846151$ $a_5 = a_3 = 21.846151$ $a_4 = 25.688356$

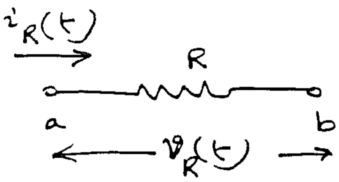
ENGR 370

Tutorial II

(1) Obtain the models of the following components in the Laplace-transform domain: (a) resistor, (b) inductor, (c) capacitor.

Solution:

(a) Resistor:

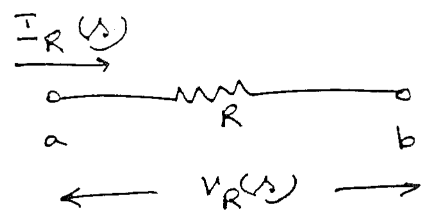


we have $v_R(t) = R \cdot i_R(t)$.

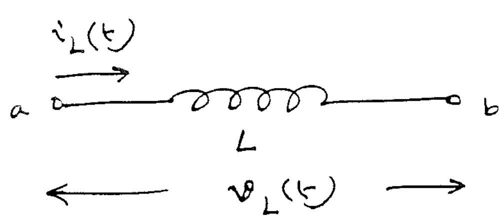
Taking Laplace transforms on both sides,

$$V_R(s) = R \cdot I_R(s)$$

The equivalent circuit in the transform domain will be



(b) Inductor:



we have $v_L(t) = L \cdot \dfrac{d\, i_L(t)}{dt}$

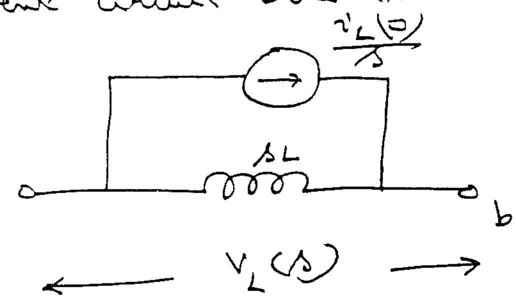
Taking Laplace transforms on both sides, we have

$$V_L(s) = L\left\{ s \cdot I_L(s) - i_L(0) \right\}$$

which can be rearranged as

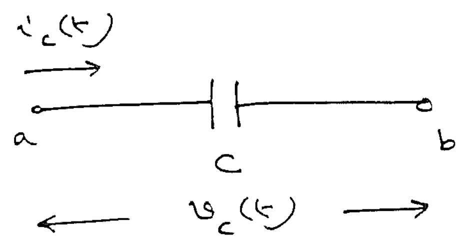
$$I_L(s) = \frac{V_L(s)}{sL} + \frac{i_L(0^+)}{s}.$$

The equivalent circuit becomes



The impedance of L is sL.

ⓒ Capacitor:



we have $\quad v_c(t) = \frac{1}{C}\int i_c(t)\,dt \quad$ or $\quad i_c(t) = C\,\frac{dv_c(t)}{dt}.$

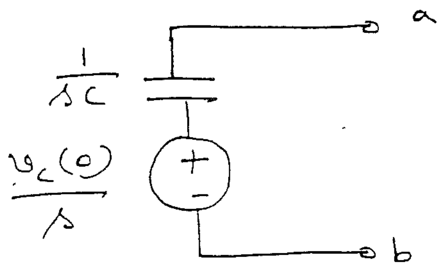
Taking Laplace transforms on both sides, we have.

$$I_c(s) = C\left[s\,V_c(s) - v_c(0)\right]$$

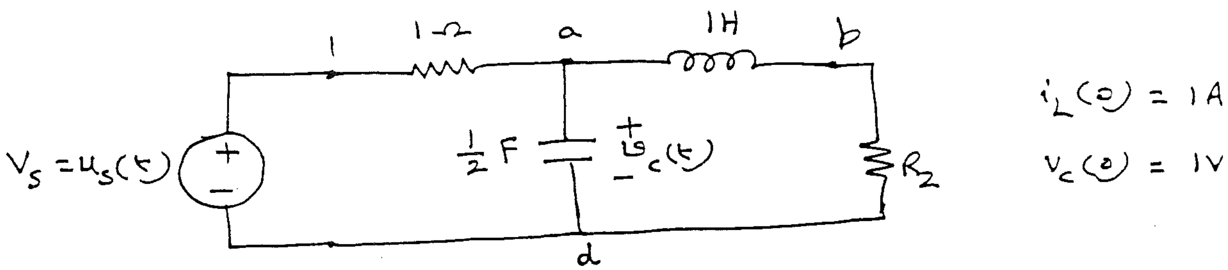
which can be rearranged as

$$V_c(s) = \frac{I_c(s)}{sC} + \frac{v_c(0^+)}{s}$$

The equivalent circuit becomes



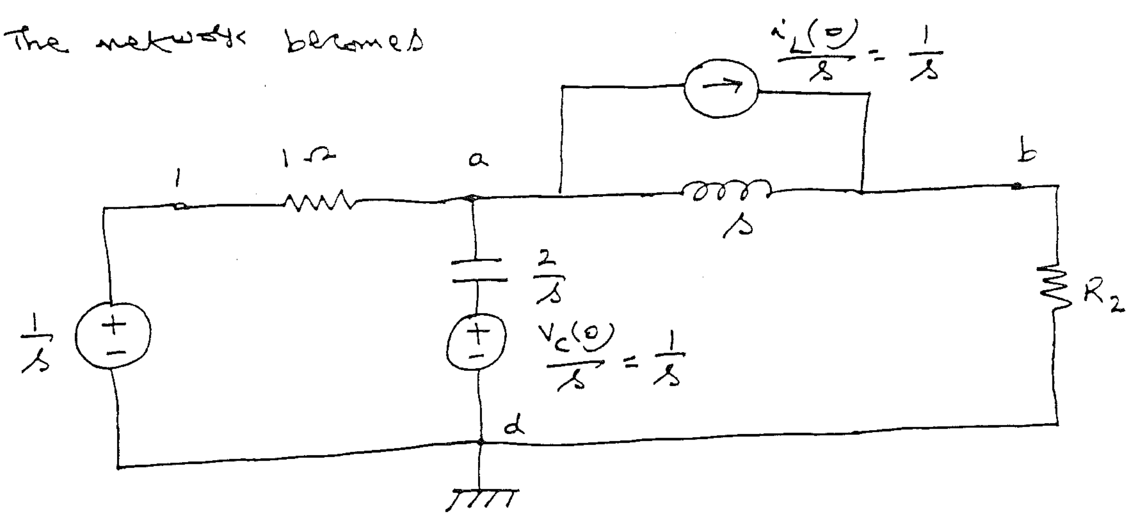
(2) For the network shown below, obtain $i_L(t)$ and $v_c(t)$ for the values (a) $R_2 = 1\Omega$, (b) $R_2 = 1.4867555\Omega$, and (c) $R_2 = 2\Omega$. Use Laplace transforms.



Solution :

Step 1 : obtain the equivalent circuit in the Laplace transform domain. For each element, substitute its model.

The network becomes



Step 2 : use known network analysis techniques to analyze the same.

Nodal analysis gives

$$V_1 = \frac{1}{s}$$

Node a:
$$\frac{V_a - V_1}{1} + \frac{V_a - \frac{v_c(0)}{s}}{\frac{2}{s}} + \frac{V_a - V_b}{s} + \frac{i_L(0)}{s} = 0$$

Node b: $\dfrac{V_b}{R_2} + \dfrac{V_b - V_a}{s} - \dfrac{i_L(0)}{s} = 0.$

The two equations can be rearranged as follows:

$$V_a\left(1 + \frac{s}{2} + \frac{1}{s}\right) - V_b \cdot \frac{1}{s} = \frac{1}{2}$$

and
$$-\frac{V_a}{s} + V_b\left(\frac{1}{s} + \frac{1}{R_2}\right) = \frac{1}{s}.$$

Solving for V_a and V_b, we get

$$V_a = \frac{s^2 + sR_2 + 2R_2}{s\left[s^2 + s(2 + R_2) + 2(R_2 + 1)\right]}$$

and
$$V_b = \frac{(s^2 + 3s + 2)R_2}{s\left[s^2 + s(2 + R_2) + 2(R_2 + 1)\right]}.$$

Case (a): $R_2 = 1\Omega$. The characteristic equation is $s^2 + 3s + 4 = 0$.

The characteristic roots are $-1.5 \pm j\,1.322288$

This corresponds to underdamped response.

$$V_a(s) = \frac{s^2 + s + 2}{s\left[(s + 1.5)^2 + (1.322288)^2\right]}$$

$$= \frac{A_1}{s} + \frac{A_2 s + A_3}{(s + 1.5)^2 + (1.322288)^2}$$

we have $(s^2 + s + 2) = A_1\left[(s + 1.5)^2 + (1.322288)^2\right] + (A_2 s + A_3)s$

Put $s = 0$. $A_1 = \frac{1}{2}$

Comparing coefficients of s^2, we get $A_1 + A_2 = 1$ or $A_2 = \frac{1}{2}$

Put $s = 1$. we get $4 = \frac{1}{2}[8] + A_2 + A_3$ or $A_3 = -\frac{1}{2}$

Therefore,

$$V_a(s) = \frac{\frac{1}{2}}{s} + \frac{\frac{1}{2}s - \frac{1}{2}}{(s+1.5)^2 + (1.32288)^2}$$

$$= \frac{\frac{1}{2}}{s} + \frac{\frac{1}{2}(s+1.5)}{(s+1.5)^2 + (1.32288)^2} + \frac{(-1.25)\frac{1}{(1.32288)}\cdot(1.32288)}{(s+1.5)^2 + (1.32288)^2}$$

$$\left\{ \text{note: } L\left[e^{-at}\cos kt\right] = \frac{s+a}{(s+a)^2 + k^2} \right.$$

$$\left. L\left[e^{-at}\sin kt\right] = \frac{k}{(s+a)^2 + k^2} \right\}$$

Hence, $v_a(t) = 0.5 + 0.5\, e^{-1.5t}\cos(1.32288t)$

$$- 0.944908\, e^{-1.5t}\sin(1.32288t).$$

$$V_b(s) = \frac{s^2 + 3s + 2}{s\left[(s+1.5)^2 + (1.32288)^2\right]}$$

$$= \frac{B_1}{s} + \frac{B_2 s + B_3}{(s+1.5)^2 + (1.32288)^2}$$

we have $s^2 + 3s + 2 = B_1\left[(s+1.5)^2 + (1.32288)^2\right] + \left[B_2 s + B_3\right]s$

Put $s=0$. $B_1 = \frac{1}{2}$

comparing coefficients of s^2, we get $B_1 + B_2 = 1$ or $B_2 = \frac{1}{2}$

put $s=1$, $6 = 8B_1 + B_2 + B_3$ or $B_3 = +\frac{3}{2}$

This gives

$$V_b(s) = \frac{\frac{1}{2}}{s} + \frac{\frac{1}{2}(s+1.5)}{(s+1.5)^2 + (1.32288)^2} + \frac{(0.75)\frac{(1.32288)}{(1.32288)}}{(s+1.5)^2 + (1.32288)^2}$$

or $v_b(t) = i_L(t)$

$$= \frac{1}{2} + \frac{1}{2} e^{-1.5t} \cos(1.32288t) + 0.566945 e^{-1.5t} \sin(1.32288$$

Case(b): $R_2 = 4.828\Omega$.

The characteristic equation is $(s+3.414)^2$.

This corresponds to critically damped response.

$$V_a(s) = \frac{s^2 + 3.414s + 6.828}{s(s+3.414)^2}$$

$$= \frac{A_0}{s} + \frac{A_1}{s+3.414} + \frac{A_2}{(s+3.414)^2}$$

This gives

$$(s^2 + 3.414s + 6.828) = A_0(s+3.414)^2 + A_1 s(s+3.414) + A_2 s.$$

Put $s=0$. $A_0 = 0.5858$

Put $s = -3.414$. $A_2 = -2$

Comparing the coefficients of s^2, we have $A_0 + A_1 = 1$ or $A_1 = 0.4142$

This yields $v_a(t) = 0.5858 + 0.4142 e^{-3.414t} - 2t e^{-3.414t}$.

$$V_b(s) = \frac{4.828(s^2+3s+2)}{s(s+3.414)^2} = \frac{B_0}{s} + \frac{B_1}{s+3.414} + \frac{B_2}{(s+3.414)^2}$$

This gives

$$4.828(s^2+3s+2) = B_0(s+3.414)^2 + B_1 s(s+3.414) + B_2 s.$$

Put $s=0$. $B_0 = 0.8285$

Put $s = -3.414$. $B_2 = -4.828$

Comparing coefficients of s^2, we have $B_0 + B_1 = 1$ or $B_1 = 0.1714$

//

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This yields

$$v_b(t) = 0.8285 + 0.1714\, e^{-3.414t} - 4.828\, t\, e^{-3.414t}$$

Case (c): $R_2 = 6\Omega$.

The characteristic equation is $s^2 + 8s + 14 = 0$.

The characteristic roots are -2.586 or -5.414

$$V_a(s) = \frac{s^2 + 6s + 12}{s(s+2.586)(s+5.414)} = \frac{A_1}{s} + \frac{A_2}{s+2.586} + \frac{A_3}{s+5.414}$$

or $$(s^2 + 6s + 12) = A_1(s+2.586)(s+5.414) + A_2 s(s+5.414) + A_3 s(s+2.586).$$

put $s = 0$. $A_1 = 0.8571$

put $s = -2.586$. $A_2 = -0.4282$

put $s = 5.414$. $A_3 = 0.5765$

we get $$v_a(t) = 0.8571 + 0.5765\, e^{-5.414t} - 0.4282\, e^{-2.586t}.$$

$$V_b(s) = \frac{6(s^2 + 3s + 2)}{s(s+2.586)(s+5.414)} = \frac{B_1}{s} + \frac{B_2}{s+2.586} + \frac{B_3}{s+5.414}$$

or $$6(s^2 + 3s + 2) = B_1(s+2.586)(s+5.414) + B_2 s(s+5.414) + B_3 s(s+2.586).$$

put $s = 0$. $B_1 = 0.8574$

put $s = -2.586$ $B_2 = -0.7626$

put $s = -5.414$ $B_3 = 5.9052$

we get $$v_b(t) = 0.8574 - 0.7626\, e^{-2.586t} + 5.9052\, e^{-5.414t}$$

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