

ENGR 311 Final Exam
Summer 2009
Pierre Gauthier

1. Solve the following system of differential equations using Laplace Transforms

$$\begin{aligned}x' - 6x + 3y &= 8e^t \\ -2x + y' - y &= 4e^t\end{aligned}$$

$$\begin{aligned}x(0) &= 1 \\ y(0) &= 0\end{aligned}$$

2. Given

$$f(x) = \begin{cases} \pi & -\pi \leq x \leq 0 \\ x & 0 \leq x \leq \pi \end{cases}$$

- Find the Fourier Series
- Sketch the graph from $-7\pi, 7\pi$
- Evaluate the sum

$$\sum_{n=1}^{\infty} \frac{(-1)^n - 1}{\pi n^2}$$

3. Solve the following wave equation

$$\frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 U}{\partial x^2}$$

With the conditions

$$\begin{aligned}U(0, t) &= U(1, t) = 0 \\ U(x, 0) &= 0 \\ \frac{\partial U}{\partial x}(x, 0) &= 2 \sin\left(\frac{3\pi}{4}x\right) - \frac{1}{4} \sin(2\pi x)\end{aligned}$$

4. a) Solve the following Heat equation

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2} - 3U$$

With the conditions

$$\begin{aligned}\frac{\partial U}{\partial x}(0, t) &= \frac{\partial U}{\partial x}(1, t) = 0 \\ U(x, 0) &= x(1 - x)\end{aligned}$$

b) What is the steady-state temperature?

5. Solve the following Laplace Equation

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$$

With the conditions

$$\begin{aligned}U(0, y) &= F(y) \text{ and } U(b, y) = G(y) \\ U(x, 0) &= U(x, a) = 0\end{aligned}$$