Concordia University
Department of Mechanical and Industrial Engineering
Final Examination

Course:	ENGR 311 , Sections X and T.
Date:	December 17^{th}, 2003.
Given by:	Dr. Pierre Q. Gauthier and Dr. Ashok Kaushal.
Instructions:	Answer all questions.
	Only non-programmable calculators are permitted.

1. Solve the following system using Laplace transforms

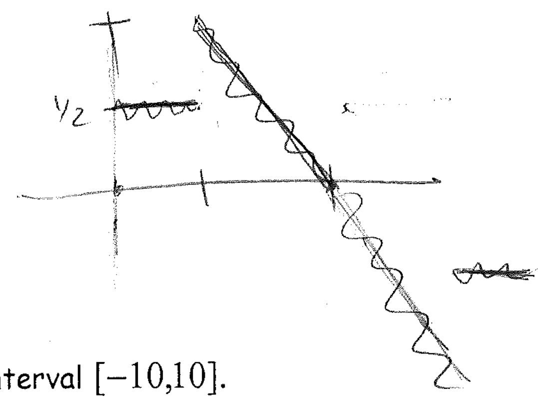
[15]

$$x'' + y' = \cos(t)$$
$$y'' - x = \sin(t)$$
$$x(0) = x'(0) = -1,\; y(0) = 1 \text{ and } y'(0) = 0$$

2. Solve,

[15] a) $\text{£}^{-1}\left\{\dfrac{2s+6}{(s^2+6s+10)^2}\right\}$ b) $y(t) + 2\displaystyle\int_0^t y(u)\cos(t-u)\,du = t$

3. Consider the following function:



$$f(x) = \begin{cases} \frac{1}{2} & , 0 \le x \le 1 \\ 2 - x & , 1 \le x \le 2 \end{cases}$$

[15]

a) Find the Fourier Sine Series of $f(x)$ and sketch it on the interval $[-10,10]$.
b) To what values will this series converge at x = -6.7, x = 3.3 and x = 8?
c) **WITHOUT FINDING THE SERIES** sketch the Fourier Cosine Series of $f(x)$, on a separate graph, also on the interval $[-10,10]$.

4. Determine constants c_1, c_2 and c_3 so that the following functions:

$$f_1(x) = 1,\; f_2(x) = c_1 + c_2 x \text{ and } f_3(x) = c_3 x^2 - \tfrac{1}{2}$$

[10]

form an orthogonal set on the interval: $-1 \le x \le 1$.

5. a) Solve, using the method of separation of variables, the following Wave equation:

$$\frac{\partial^2 u}{\partial t^2} + \beta \frac{\partial u}{\partial t} = \frac{1}{4} \frac{\partial^2 u}{\partial x^2}$$

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[15]

with fixed boundary conditions: $u(0,t) = u(1,t) = 0$, and with initial conditions:

$$u(x,0) = 0 \text{ and } \frac{\partial u(x,0)}{\partial t} = \frac{1}{2}\sin(\pi x) - \frac{1}{3}\sin(2\pi x).$$

b) What is the physical significance of the term: $\beta \frac{\partial u}{\partial t}$? What sign must β have?

c) How does the solution behave if $\beta = 0$? What if $\beta \neq 0$?

6. Solve, using the method of separation of variables, the following Heat equation:

$$\frac{\partial u}{\partial t} = 9 \frac{\partial^2 u}{\partial x^2}$$

[15]

with adiabatic boundary conditions: $\frac{\partial u(0,t)}{\partial x} = \frac{\partial u(3,t)}{\partial x} = 0$, and with an initial temperature distribution:

$$u(x,0) = \begin{cases} 2 & ,0 \le x \le 1 \\ 3 - x & ,1 \le x \le 3 \end{cases}$$

What is the steady state temperature on $0 \le x \le 3$?

7. Find the Steady-State temperature distribution on a plate the shape of a quarter circle, with radius 5 cm, by solving Laplace's equation:

$$\nabla^2 u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

[15]

with the temperatures on the edges given by:

$$u(5,\theta) = \frac{1}{2} - \frac{\theta}{\pi}, \quad 0 \le \theta \le \frac{\pi}{2} \quad \text{and} \quad u(r,0) = u(r,\frac{\pi}{2}) = 0, \quad 0 \le r \le 5.$$