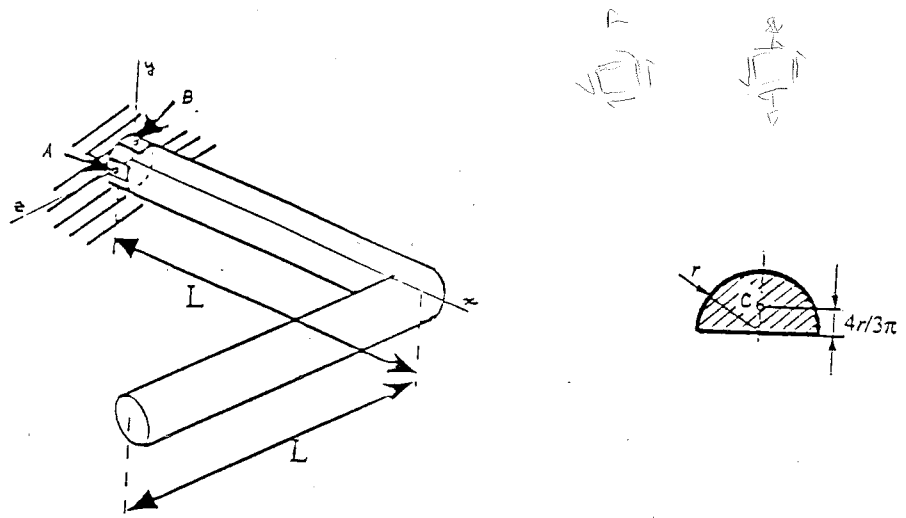


96

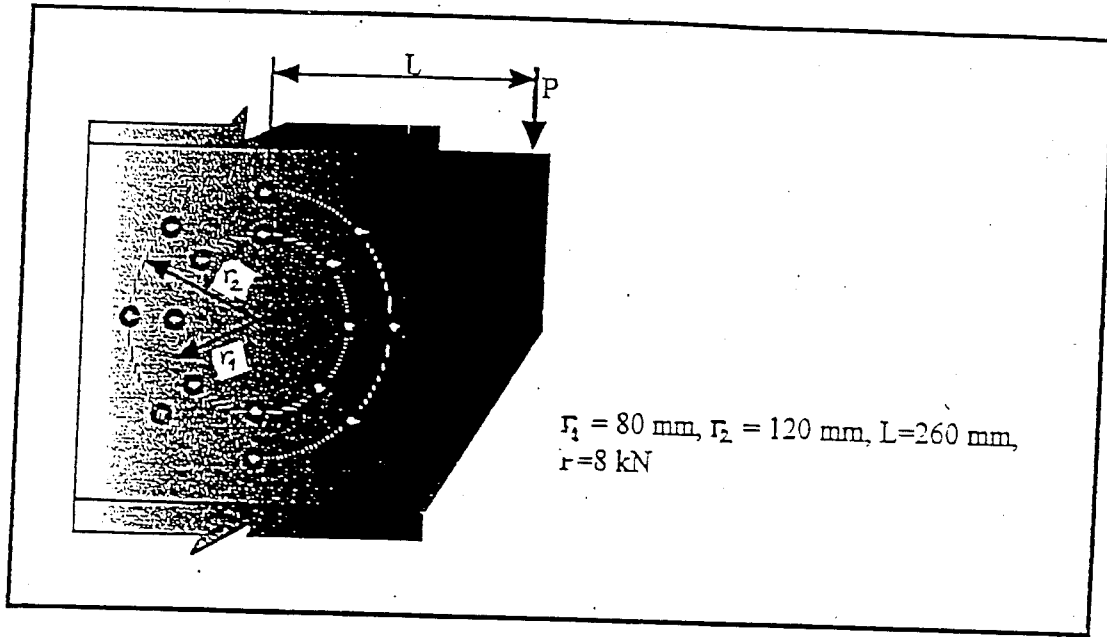
CONCORDIA UNIVERSITY
DEPARTMENT OF MECHANICAL ENGINEERING

COURSE: MECHANICS OF MATERIALS	NUMBER: ENGR 244/4	SECTION: T
EXAMINATION: MIDTERM	DATE & TIME: March 14, 2000, 13:15 - 14:30	# OF PAGES: 2
INSTRUCTOR: Dr. Eliza M. Haseganu		
MATERIAL ALLOWED:	<input checked="" type="checkbox"/> NO	<input type="checkbox"/> YES
CALCULATORS ALLOWED:	<input type="checkbox"/> NO	<input checked="" type="checkbox"/> YES
SPECIAL INSTRUCTIONS: Answer both questions. They have equal value.		

1. Determine the state of stress at points A and B due to the self weight of the bar shown below. The two sections of the bar make a right angle with each other. Assume that the bar has a radius of 40 mm, the length of each section is $L = 1$ m, and the weight density of the material of the bar is 77 kN/m^3 .



2. Calculate the shear force in the bolts that carry the maximum and minimum forces. All bolts have the same diameter of 20 mm and are made of the same material.



EXAMINATION
ENGR 244

FORMULAE

$$\tau = \frac{T\rho}{J}$$

$$\phi = \frac{TL}{GJ}$$

$$\phi = \frac{My}{I}$$

$$\tau = \frac{VQ}{I}$$

$$Eiy'' = M(x)$$

$$Eiy^{(4)} = q(x)$$

$$\sigma_{cr} = \frac{\pi^2 E}{(kL/r)^2}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{(\sigma_x - \sigma_y)}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_n = -\frac{(\sigma_x - \sigma_y)}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{p1} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{max}$$

$$\sigma_{p2} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{min}$$

$$\Delta = \frac{PL}{AE}$$

$$\sigma_1 = \sigma_2 = \frac{rP}{2t}$$

$$\sigma_h = \sigma_1 = \frac{rP}{t}$$

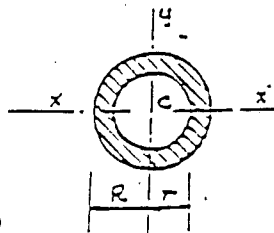
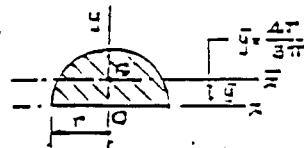
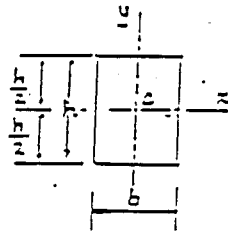
$$\tau_r = \frac{T\rho_r}{J}$$

$$F_r = \frac{T\rho_r}{\Sigma(\rho_r)^2}$$

$$q = \frac{VQ}{I}$$

$$J = \frac{\pi r^4}{2}$$

$$I_x + I_y = J$$



Rectangular Area

$$I_x = \frac{bh^3}{12}$$

area of part

$$I_x = \bar{I} + Ad^2$$

Inertia of part

center of part to centroid

Semi-Circular Area

$$I_x = I_y = \frac{1}{8} \pi r^4$$

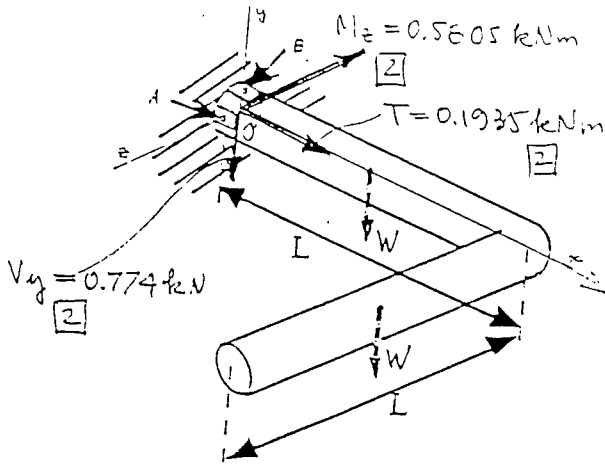
$$I_o = \frac{1}{4} \pi r^4$$

Hollow Sectional Area

$$I_x = I_y = \frac{\pi}{4} (R^4 - r^4)$$

$$I_z = I_o = \frac{\pi}{2} (R^4 - r^4)$$

ENGR 244 MIDTERM SOLUTION

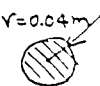


• WEIGHT W of each section:

$$W = \gamma (\pi r^2 L) = 77 (\pi (0.04)^2 (1)) = 0.387 \text{ kN} \quad [2]$$

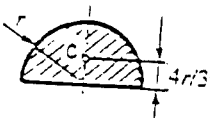
• EQUIV. FORCE-COUPLE SYST at O:

$$\begin{cases} V_y = 2W = 2(0.387) = 0.774 \text{ kN} \\ T = W \frac{L}{2} = 0.387(0.5) = 0.1935 \text{ kNm} \\ M_z = W \frac{L}{2} + WL = \frac{3WL}{2} = 0.5805 \text{ kNm} \\ M_y = 0 \end{cases}$$

• CHAR. of CROSS-SECTION: 

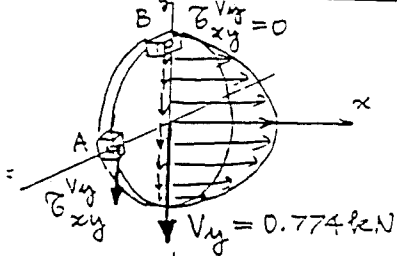
$$I = \frac{\pi r^4}{4} = \frac{\pi (0.04)^4}{4} = 2.01 (10^{-6}) \text{ m}^4$$

$$J = \frac{\pi r^4}{2} = \frac{\pi (0.04)^4}{2} = 4.021 (10^{-6}) \text{ m}^4$$

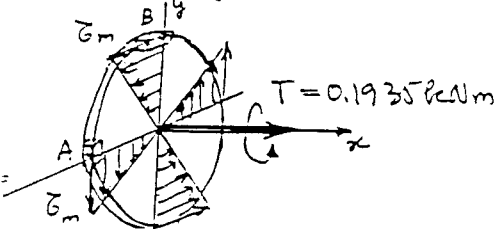


$$Q = \frac{1}{2} \pi r^2 \frac{4r}{3\pi} = \frac{4(0.04)^3}{3} = 4.267 (10^{-5}) \text{ m}^3 \quad [2]$$

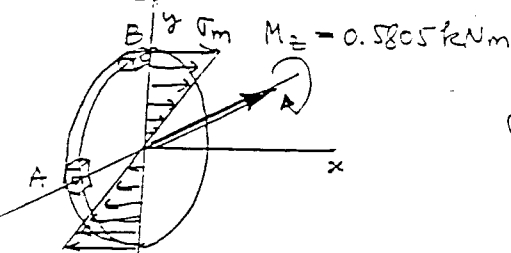
• DISTRIB. of STRESSES in CROSS SECT. at O



$$\tau_{xy} = \frac{V_y Q}{I t} = \frac{V_y Q}{I (2r)} = \frac{0.774 (4.267) (10^{-5})}{2.01 (10^{-6}) (0.08)} = 205.37 \text{ kPa} \quad [6]$$

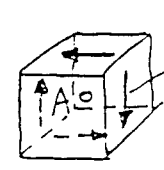


$$\tau_m = \frac{T c}{J} = \frac{T r}{J} = \frac{0.1935 (0.04)}{4.021 (10^{-6})} = 1924.89 \text{ kPa} \quad [6]$$

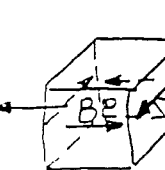


$$\sigma_m = \frac{M_z c}{I} = \frac{M_z r}{I} = \frac{0.5805 (0.04)}{2.01 (10^{-6})} = 11552.24 \text{ kPa} \quad [6]$$

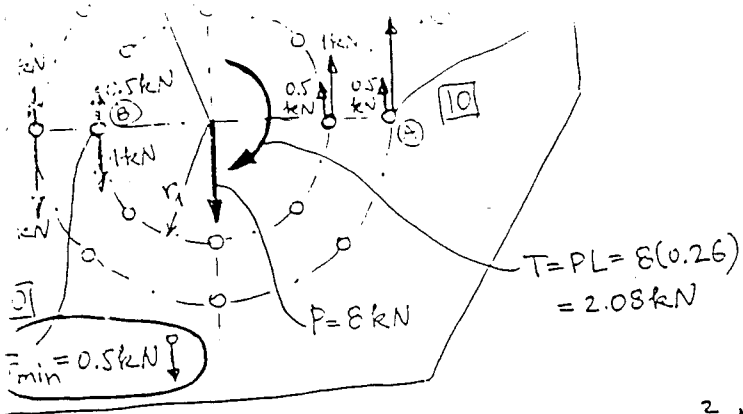
• STATE of STRESS at points A and B:



$$\begin{aligned} \tau_{xy} &= \tau_{xy} + \tau_{xy}^T \\ &= 205.37 + 1924.89 \\ &= 2130.26 \text{ kPa} \end{aligned} \quad [10]$$



$$\begin{aligned} \sigma_x &= \sigma_m = 11552.24 \text{ kPa} \\ \tau_{xz} &= 1924.89 \text{ kPa} \end{aligned} \quad [10]$$



all bolts have the same cross sectional area A

• SHEAR STRESS (due to torque T):

in BOLT i at $s_i = r_1 = 0.08 \text{ m}$

$$\tau_{t,i} = \frac{T r_1}{8A[r_1^2 + r_2^2]}$$

in BOLT i at $s_i = r_2 = 0.12 \text{ m}$

$$\tau_{t,i} = \frac{T r_2}{8A[r_1^2 + r_2^2]}$$

• SHEAR FORCE (due to torque T):

in BOLT i at $s_i = r_1 = 0.08 \text{ m}$

$$F_{t,i} = \left(\tau_{t,i} \right) A = \frac{T r_1}{8[r_1^2 + r_2^2]} = \frac{2.08(0.08)}{8(0.08^2 + 0.12^2)} = \underline{1 \text{ kN}} \quad \left(\perp \text{ radius } r_1 \text{ at bolt } i \right)$$

in BOLT i at $s_i = r_2 = 0.12 \text{ m}$

$$F_{t,i} = A \tau_{t,i} = \frac{T r_2}{8(r_1^2 + r_2^2)} = \frac{2.08(0.12)}{8(0.08^2 + 0.12^2)} = \underline{1.5 \text{ kN}} \quad \left(\perp \text{ radius } r_2 \text{ at bolt } i \right)$$

■ MAXIMUM SHEAR FORCE

in BOLT (A) at $s_i = r_2$

$$F_{\max} = 0.5 + 1.5 = \underline{2 \text{ kN}} \uparrow$$

(maximum shear stress, not required $\tau_{\max} = \frac{F_{\max}}{A}$)

■ MINIMUM SHEAR FORCE

in BOLT (B) at $s_i = r_1$

$$F_{\min} = 1 - 0.5 = \underline{0.5 \text{ kN}} \downarrow$$

(minimum shear stress, not required $\tau_{\min} = \frac{F_{\min}}{A}$)

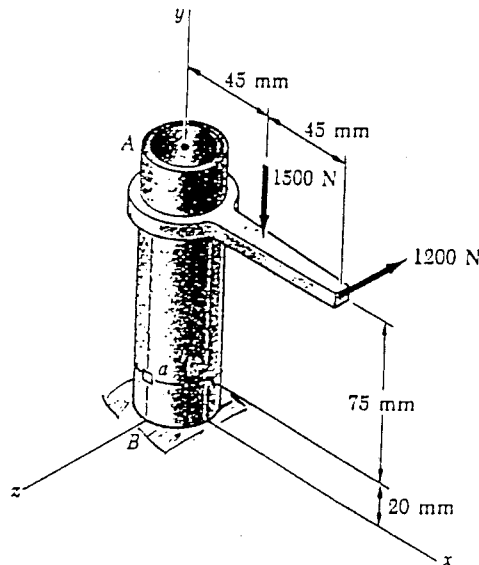
Σ 50

Total: 100

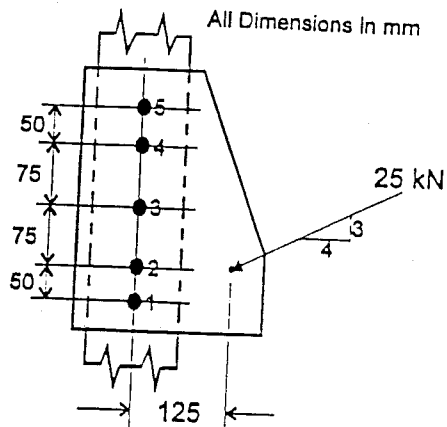
CONCORDIA UNIVERSITY
DEPARTMENT OF MECHANICAL ENGINEERING

COURSE: MECHANICS OF MATERIALS	NUMBER: ENGR 244/4	SECTION: T
EXAMINATION: MIDTERM	DATE & TIME: March 6, 2001, 13:15 – 14:30	# OF PAGES: 2
INSTRUCTOR: Dr. Eliza M. Haseganu		
MATERIAL ALLOWED:	<input checked="" type="checkbox"/> NO	<input type="checkbox"/> YES
CALCULATORS ALLOWED:	<input type="checkbox"/> NO	<input checked="" type="checkbox"/> YES
SPECIAL INSTRUCTIONS: Answer both questions. They have equal value.		

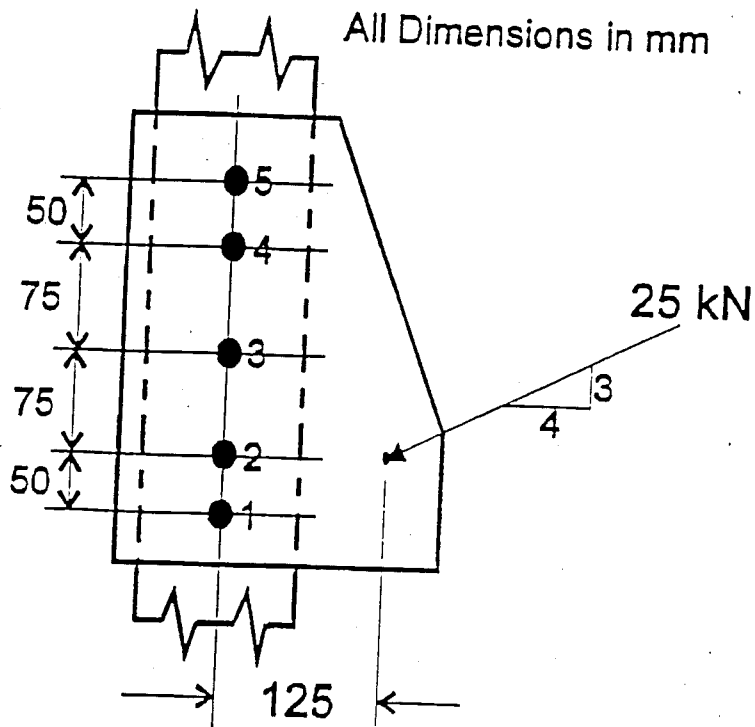
1. Two forces are applied to the pipe AB as shown. Knowing that the pipe has inner and outer diameters equal to 35 and 42 mm, respectively, determine the normal and shear stresses at point a .



2. Each of the bolts in the joint shown has a diameter of 15 mm.
- Determine the magnitude of the maximum stress in the bolts.
 - Compute the maximum bearing stress if the gusset plate is 10 mm thick.



SOLUTION



7

EXAMINATION
ENGR 244

FORMULAE

$$\tau = \frac{T\rho}{J}$$

$$\phi = \frac{TL}{GJ}$$

$$\sigma = \frac{My}{I}$$

$$\tau = \frac{VQ}{It}$$

$$EIy'' = M(x)$$

$$EIy'''' = q(x)$$

$$\sigma_{cr} = \frac{\pi^2 E}{(kL/r)^2}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{(\sigma_x - \sigma_y)}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_n = -\frac{(\sigma_x - \sigma_y)}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{p1} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{max}$$

$$\sigma_{p2} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{min}$$

$$\Delta = \frac{PL}{AE}$$

$$\sigma_s = \sigma_t = \frac{rP}{2l}$$

$$\sigma_h = \sigma_l = \frac{rP}{t}$$

$$\tau_t = \frac{T\rho}{J}$$

$$F_t = \frac{T\rho}{\Sigma(\rho)^2}$$

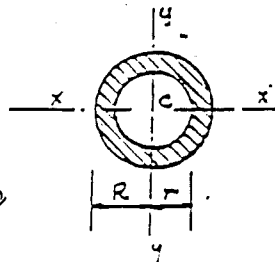
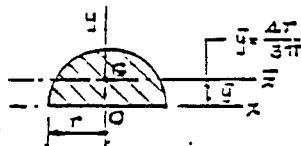
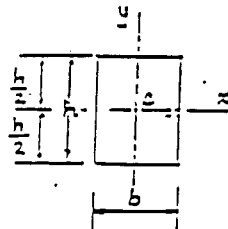
$$q = \frac{VQ}{I}$$

$$J = \frac{\pi}{2} r^4$$

$$I_x + I_y = J$$

$$\tau = \frac{P}{2\pi R}$$

$$\frac{J}{C} = \frac{T}{\tau}$$



Rectangular Area

$$I_x = \frac{bh^3}{12}$$

Semi-Circular Area

$$I_x = I_y = \frac{1}{8} \pi r^4$$

$$J_o = \frac{1}{4} \pi r^4$$

Hollow Sectional Area

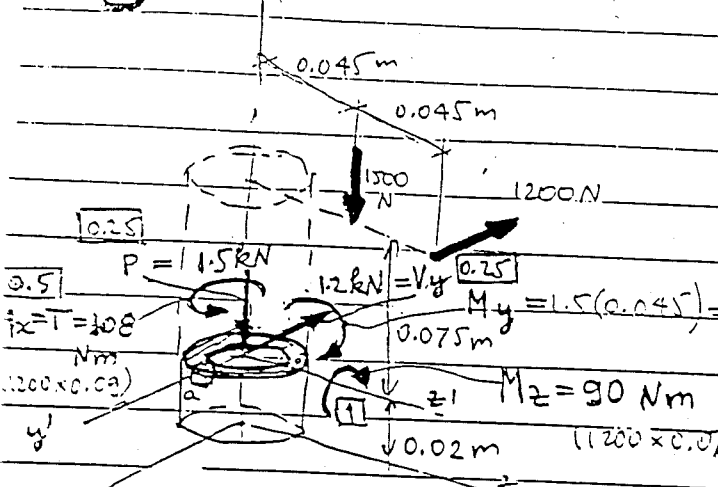
$$I_x = I_y = \frac{\pi}{4} (R^4 - r^4)$$

$$I_z = J_o = \frac{\pi}{2} (R^4 - r^4)$$

ENGR 244 MIDTERM SOLUTION

1

x (y)



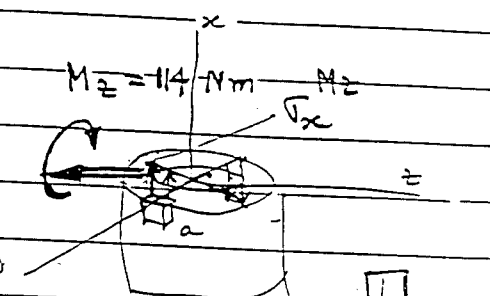
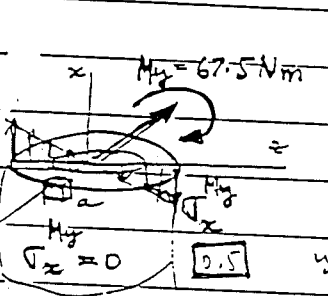
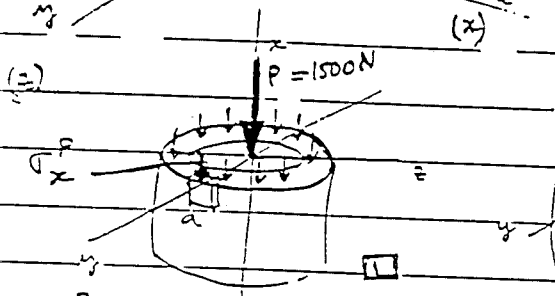
$$A = \frac{\pi}{4} (0.042^2 - 0.035^2) = 4.23 \times 10^{-4} \text{ m}^2$$

$$A = \pi (0.021^2 - 0.0175^2)$$

$$I = \frac{\pi}{4} (0.021^4 - 0.0175^4) = 7.9 \times 10^{-8} \text{ m}^4$$

$$J = \frac{\pi}{2} (0.021^4 - 0.0175^4) = 1.58 \times 10^{-7} \text{ m}^4$$

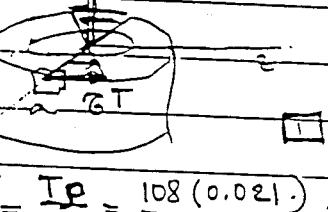
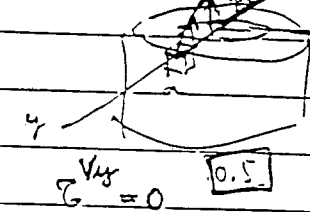
$P = 1.5 \text{ kN}$
 $1.2 \text{ kN} = V_y$
 $M_y = 1.5(0.045) = 67.5 \text{ Nm}$
 $M_z = 90 \text{ Nm}$
 $T = 108 \text{ Nm}$
 (1200×0.09)



$$\sigma_x = \frac{P}{A} = \frac{1500}{4.23 \times 10^{-4}} = 3.556 \text{ MPa}$$

$$\tau_{xy} = 0$$

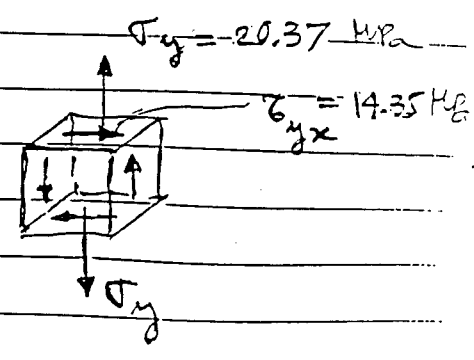
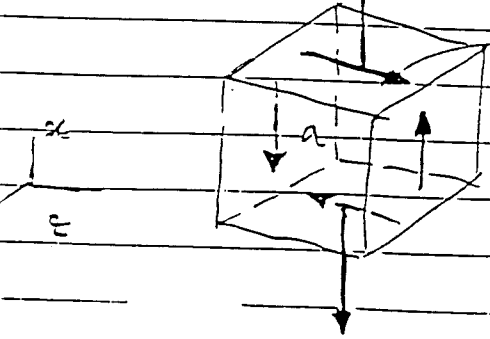
$$\sigma_x = \frac{M_z c}{I} = \frac{90(0.021)}{7.9 \times 10^{-8}} = 23.92 \text{ MPa}$$



$$\tau_{xz} = \frac{T \rho}{J} = \frac{108(0.021)}{1.58 \times 10^{-7}} = 14.35 \text{ MPa}$$

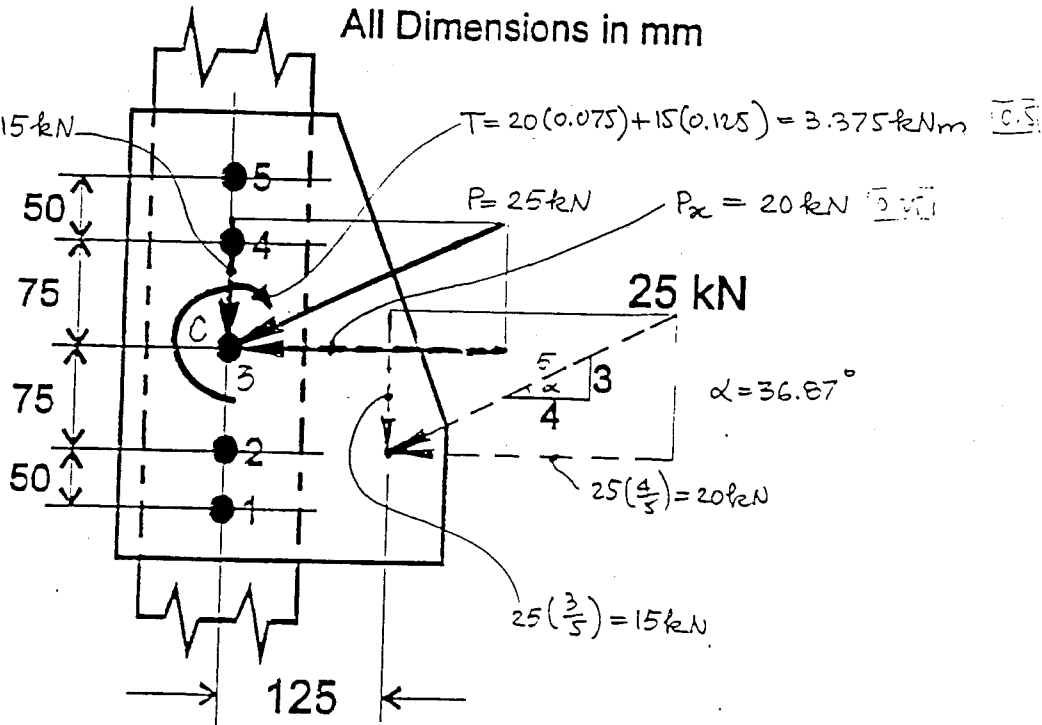
$$\sigma_x = \sigma_x - \sigma_x = 23.92 - 3.55 = 20.37 \text{ MPa}$$

$$\tau_{xz} = 14.35 \text{ MPa}$$



9

2



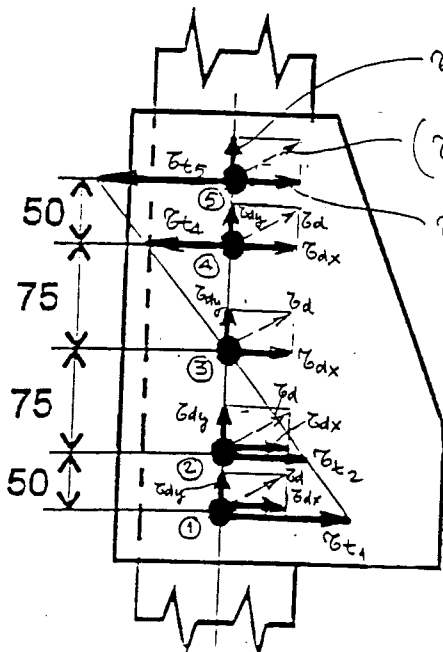
$$A_i = \frac{\pi d^2}{4} = \frac{\pi (0.015)^2}{4} = 1.7672 (10)^{-4} \text{ m}^2$$

$$\sum S_i^2 = 4.25 (10)^{-2} \text{ m}^2$$

$$J = \sum S_i^2 A_i = A_i \sum S_i^2 = 1.7672 (10)^{-4} [2(0.075)^2 + 2(0.125)^2] = 7.5106 (10)^{-6} \text{ m}^4 \quad \boxed{1}$$

(i) MAGNITUDE OF MAXIMUM STRESS in BOLTS

- DIRECT SHEAR STRESS - same in all BOLTS ① → ⑤



$$\tau_{dy} = \frac{P_y}{nA} = \frac{15}{5(1.7672)10^{-4}} = 16.976 \text{ MPa} \quad \boxed{0.5}$$

$$\tau_d = \frac{P}{nA} = \frac{25}{5(1.7672)10^{-4}} = 28.294 \text{ MPa}$$

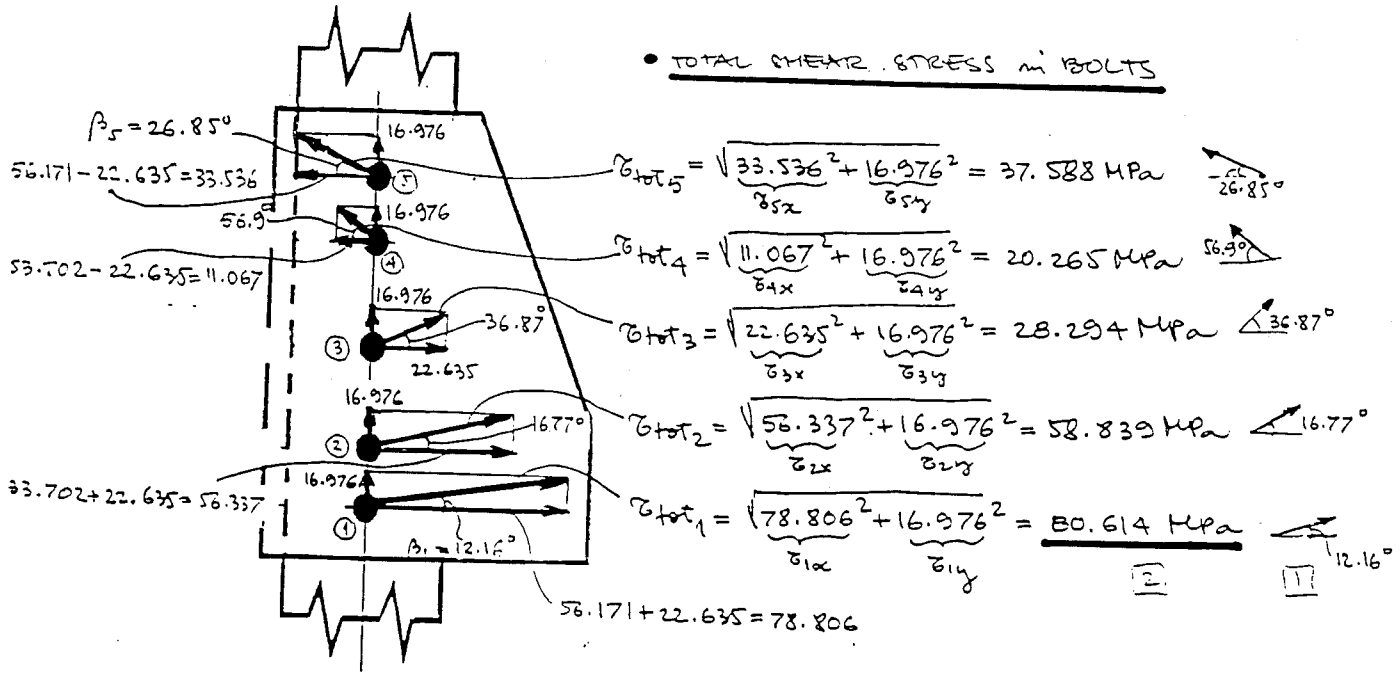
$$\tau_{dx} = \frac{P_x}{nA} = \frac{20}{5(1.7672)10^{-4}} = 22.635 \text{ MPa} \quad \boxed{0.5}$$

- SHEAR STRESS due to TORSION $\tau_{ti} = \frac{TS_i}{J}$

$$\tau_{t_{1,5}} = \frac{TS_{1,5}}{J} = \frac{3.375(0.125)}{7.5106(10)^{-6}} = 56.171 \text{ MPa} \quad \boxed{1}$$

$$\tau_{t_{2,4}} = \frac{TS_{2,4}}{J} = \frac{3.375(0.075)}{7.5106(10)^{-6}} = 33.172 \text{ MPa} \quad \boxed{1}$$

TOTAL SHEAR STRESS in BOLTS



$$\tan \beta_1 = \frac{\sigma_{1y}}{\sigma_{1x}} = \frac{16.976}{78.806} = 0.215 \Rightarrow \beta_1 = 12.16^\circ$$

$$\tan \beta_2 = \frac{\sigma_{2y}}{\sigma_{2x}} = \frac{16.976}{58.337} \Rightarrow \beta_2 = 16.77^\circ$$

$$\tan \beta_3 = \frac{\sigma_{3y}}{\sigma_{3x}} = \frac{16.976}{22.635} \Rightarrow \beta_3 = 36.87^\circ$$

$$\tan \beta_4 = \frac{\sigma_{4y}}{\sigma_{4x}} = \frac{16.976}{11.067} \Rightarrow \beta_4 = 56.9^\circ$$

$$\tan \beta_5 = \frac{\sigma_{5y}}{\sigma_{5x}} = \frac{16.976}{33.536} \Rightarrow \beta_5 = 26.85^\circ$$

(ii) MAXIMUM BEARING STRESS in GUSSET PLATE

$$\sigma_{max1} = \frac{\sigma_{tot1} A_1}{t d_1} = \frac{F_{tot1}}{t d_1} = \frac{80.614 (1.7672) 10^{-4}}{0.01 (0.015)} = 94.974 \text{ MPa} \quad \boxed{2}$$

$$\sigma_{max2} = \frac{\sigma_{tot2} A_2}{t d_2} = \frac{F_{tot2}}{t d_2} = \frac{58.839 (1.7672) 10^{-4}}{0.01 (0.015)} = 69.313 \text{ MPa}$$

$$\sigma_{max3} = \frac{\sigma_{tot3} A_3}{t d_3} = \frac{F_{tot3}}{t d_3} = \frac{28.294 (1.7672) 10^{-4}}{0.01 (0.015)} = 33.333 \text{ MPa}$$

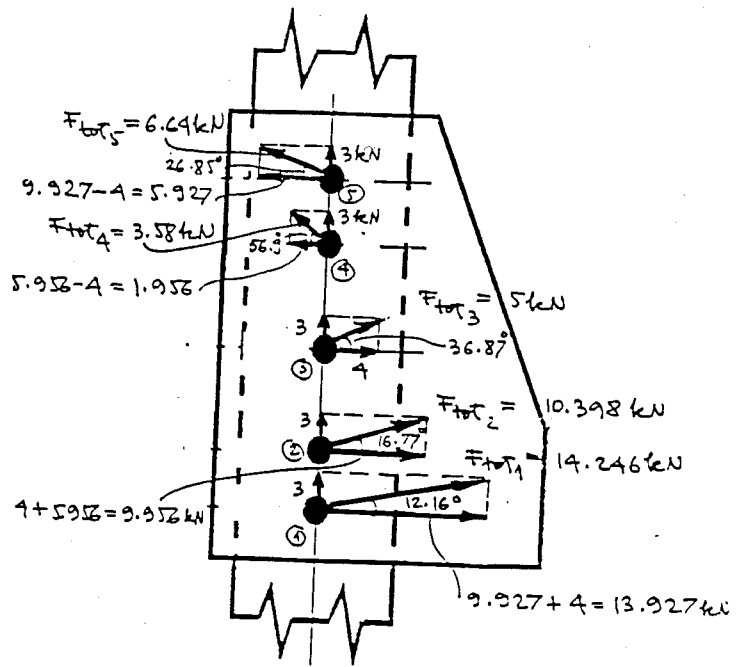
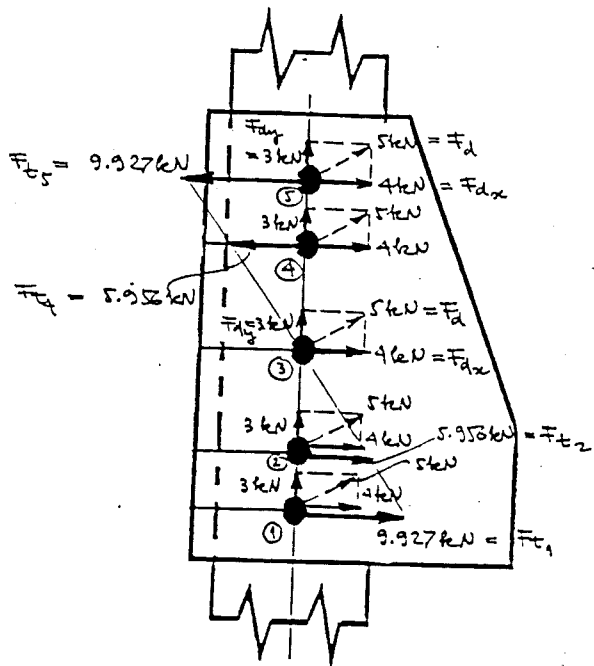
$$\sigma_{max4} = \frac{\sigma_{tot4} A_4}{t d_4} = \frac{F_{tot4}}{t d_4} = \frac{20.265 (1.7672) 10^{-4}}{0.01 (0.015)} = 23.867 \text{ MPa}$$

$$\sigma_{max5} = \frac{\sigma_{tot5} A_5}{t d_5} = \frac{F_{tot5}}{t d_5} = \frac{37.588 (1.7672) 10^{-4}}{0.01 (0.015)} = 44.267 \text{ MPa}$$

//

- END -

VERSION of SOL: Calculating TOTAL SHEAR FORCE first, and then τ_{tot}



- DIRECT SHEAR FORCE (due to centric $P = 25 \text{ kN}$)

$$F_{dx} = \frac{P_x}{n} = \frac{20}{5} = 4 \text{ kN}$$

$$F_{dy} = \frac{P_y}{n} = \frac{15}{5} = 3 \text{ kN}$$

$$(F_d = \frac{P}{n} = \frac{25}{5} = 5 \text{ kN})$$

- SHEAR FORCE due to TORQUE ($T = 3.375 \text{ kNm}$)

$$F_{t_{1,5}} = G_{t_{1,5}} A = \frac{T S_{1,5}}{(\sum S_i^2)} A = \frac{T S_{1,5}}{\sum S_i^2} = \frac{3.375 (0.125)}{4.25 (10)^{-2}} = 9.927 \text{ kN}$$

$$F_{t_{2,4}} = G_{t_{2,4}} A = \frac{T S_{2,4}}{\sum S_i^2} = \frac{3.375 (0.075)}{4.25 (10)^{-2}} = 5.958 \text{ kN}$$

- TOTAL SHEAR FORCE

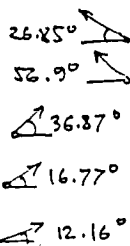
$$F_{tot1} = \sqrt{13.927^2 + 3^2} = 14.246 \text{ kN}$$

$$F_{tot2} = \sqrt{9.958^2 + 3^2} = 10.398 \text{ kN}$$

$$F_{tot3} = \sqrt{4^2 + 3^2} = 5 \text{ kN}$$

$$F_{tot4} = \sqrt{1.958^2 + 3^2} = 3.58 \text{ kN}$$

$$F_{tot5} = \sqrt{5.927^2 + 3^2} = 6.64 \text{ kN}$$



TOTAL SHEAR STRESS

$$\tau_{tot_i} = \frac{F_{tot_i}}{A_i}$$

(same results as on previous page)