

**Engineering Differential Equations**  
**Section J**  
**Exam II (A)**

**Directions:** You have **60 minutes** to solve the following 4 problems. You may use a calculator. No cell phones are allowed during the exam.

- (1) (10 points) Solve the equation

$$xy' - y = 4xy^2, \quad x > 0.$$

- (2) (10 points) A tank filled to capacity contains 200 liters of fluid in which 30 grams of salt is dissolved. Pure water is then pumped into the tank at a rate of 4 L/min; the well-mixed solution is pumped out at a rate of 5 L/min. Answer the following questions, giving the exact value calculated with this model, even if you also approximate your answer.

- (a) Find the number of grams of salt in the tank after 5 minutes.
- (b) How long does it take the tank to empty?
- (c) What is the number of grams of salt present in the brine when the tank is half full?

- (3) (10 points)

- (a) State, without explaining your choice, whether the following statement is TRUE or FALSE: “The maximum number of linearly independent solutions of an equation of the form  $y^{(n)} + a_{n-1}y^{(n-1)} + \dots + a_1y' + a_0y = 0$ , with  $a_i \in \mathbb{R}$ ,  $n \geq 2$ , is  $n$ .”
- (b) State, without explaining your choice, whether the following statement is TRUE or FALSE: “If  $y_1(x) = \sin(x)$  is a solution of a homogeneous second-order differential equation with constant real coefficients, then  $y_2(x) = \cos(x)$  is also a solution of this equation.”
- (c) Determine whether the set of functions  $f_1(x) = x$ ,  $f_2(x) = \ln(x)$ ,  $f_3(x) = \ln(x^2)$  are linearly dependent or linearly independent on the interval  $(0, \infty)$ .
- (d) Solve the initial value problem  $y'' - y' = 0$ ,  $y(0) = 1$ ,  $y'(0) = -1$ .

- (4) (10 points) Find the general solution of the differential equation

$$y'' + 5y' + 4y = 24e^{-4x}.$$