

Engineering Differential Equations
Section J
Exam I (B)

Directions: You have **60 minutes** to solve the following **4** problems. You may use a calculator. No cell phones are allowed during the exam.

- (1) (10 points) For each of the following first-order differential equations, state whether it is separable, linear, homogeneous, or none of the above. You do not need to explain your answers.

Each equation may correspond to more than one listed type (in which case you must list **all** types) or none of those.

(a) $\frac{dy}{dx} = (x - 1)(y - 1)$

(b) $y' = y\sqrt{1 - \frac{y}{x}}$

(c) $\frac{dy}{dx} = y\sqrt{1 - x^2}$

(d) $x + 3y - xy' = 0$

(e) $x \frac{dy}{dx} = \sqrt{x^2 - y^2}$

- (2) (10 points) Find the general solution of the first-order linear equation

$$\frac{dy}{dx} + 2y = e^{2x}.$$

- (3) Let

$$\left(\sqrt{x} + \frac{y}{x}\right) dx + (y^2 + \ln(2x)) dy = 0, \quad x > 0.$$

- (a) (5 points) Check that this is an exact differential equation.
(b) (5 points) Solve the equation leaving the general solution in implicit form.

- (4) Suppose that $P(t)$ (with t in months), the fish population in a lake contaminated by chemicals, satisfies the differential equation

$$\frac{dP}{dt} = -kP, \quad k > 0.$$

- (a) (5 points) Find the general solution of this differential equation.
(b) (5 points) If today there are 400 fish in the pond, and we know that in a month there will be 300 fish in the pond, how many will be there after 1 year? (Give the exact value calculated with this model, do not approximate.)