Concordia University
Electrical & Computer Engineering
Final Exam, Winter 2000
EMAT 332 Vector Calculus and Partial Differential Equations
Instructor: R. Paknys

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i) No calculators
ii) Closed book exam, no notes or crib sheets
iii) Permitted materials: pen, pencil, eraser, ruler

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(1) Directly evaluate the integral

$$I = \int_C \hat{z}(y + y^2) \cdot d\vec{\ell}$$

where C is the perimeter of the triangle shown.

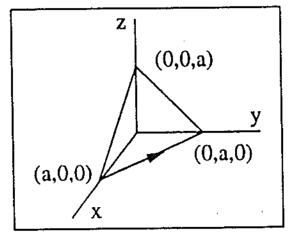

Figure 1: For problems 1,2.

(2) Solve Problem 1 again, this time using Stokes' theorem.

(3) For a bar of length L, solve the heat equation for $u(x,t)$

$$\frac{\partial u}{\partial t} = \kappa \frac{\partial^2 u}{\partial x^2}$$

subject to the boundary conditions $\partial u(0,t)/\partial x = 0$, and $u(L,t) = 0$, and the initial condition $u(x,0) = u_0$ where u_0 is a constant. Sketch the solution at $t = 0$ for $-5L \leq x \leq 5L$, showing the periodicity in x.

(4) Solve Laplace's equation for $u(\rho, \phi)$ in cylindrical coordinates

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho \frac{\partial u}{\partial \rho}) + \frac{1}{\rho^2}\frac{\partial^2 u}{\partial \phi^2} = 0$$

in the quarter disk region R, as shown. The conditions are $|u| < \infty$ for all $(\rho, \phi) \in R$, $u(a, \phi) = u_0$ on $0 \leq \phi \leq \pi/2$, where u_0 is a constant, and $u(\rho, 0) = 0$, $u(\rho, \pi/2) = 0$. Sketch the solution at $\rho = a$ for $0 \leq \phi \leq 2\pi$, showing the periodicity in ϕ.

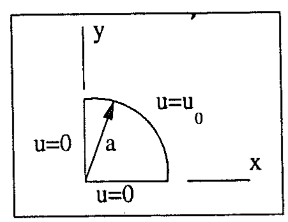

Figure 2: For problem 4.

(5) Expand the function $f(\rho) = 1$ on $0 \leq \rho \leq a$ by using a Fourier-Bessel series

$$f(\rho) = \sum_{k=1}^{\infty} A_k J_0(\alpha_k \rho / a)$$

where the α_k are defined by $J_0(\alpha_k) = 0$.

Orthogonality of Bessel Functions: With α_k given by $J_n(\alpha_k) = 0$, we have

$$\int_0^1 x J_n(\alpha_i x) J_n(\alpha_j x) dx = \begin{pmatrix} 0; \; i \neq j \\ \frac{1}{2} J_n'^2(\alpha_i); \; i = j \end{pmatrix}$$

Recursion Relations:

$$J_0'(x) = -J_1(x)$$

$$J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x)$$

$$J_n'(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$$

$$x J_n'(x) = x J_{n-1}(x) - n J_n(x)$$

$$x J_n'(x) = n J_n(x) - x J_{n+1}(x)$$

$$[x^n J_n(x)]' = x^n J_{n-1}(x)$$

$$[x^{-n} J_n(x)]' = -x^{-n} J_{n+1}(x)$$

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Electrical & Computer Engineering
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Instructor: M. Danial

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1) a) Using arrows of proper magnitude and direction, sketch the vector function:

$$\vec{F}(x,y) = i + jy$$

b) Write a formula for a vector function in two dimentions whose direction is tangential and whose magnitude at any point *(x,y)* is equal to its distance from the origin.

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2) Evaluate the surface integral $\iint_S G(x,y,z)\, ds$

where $G(x,y,z) = \dfrac{1}{1 + 4\,(x^2 + y^2)}$

where *S* is the portion of the paraboloid $z = x^2 + y^2$ between $z = 0$ and $z = 1$

note that the final integral can be done by using polar coordinates.

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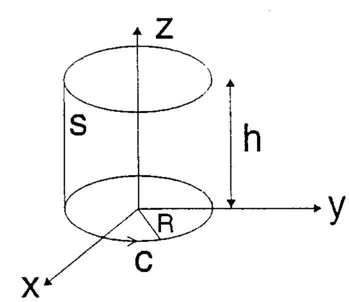
3) Evaluate the line integral $I = \int_c (2yi - 2xj + 2zk) \cdot dl$



where *c*, the circle of radius *R* lying, in the *xy* plane, centered at (0, 0, 0) and directed as shown in figure, or use stokes' theorem.

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4) Apply the separation of variables technique to solve the partial diferential equation:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2} \quad , \quad \frac{\partial U}{\partial x}(0, t) = 0 \quad , \quad U(2, t) = 0$$

or use Frobenius method

$$U(x, 0) = 8 \cos \frac{2\pi x}{4} - 6 \cos \frac{9\pi x}{4}$$

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(1) (a) Using arrows of proper magnitude and direction, sketch the vector function

$$\vec{A}(x,y) = \hat{x}y + \hat{y}xy$$

for $x = 1$, $-2 \le y \le 2$. (b) Convert $\vec{A}$ to cylindrical coordinates. (c) Write a formula for a vector function in two dimensions where the direction is tangential (in the sense as shown in Figure 1) and whose magnitude at any point (x, y) is equal to the distance from the origin. Give the final result in rectangular coordinates. (5 marks)

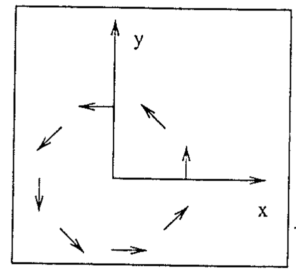

Figure 1: For problem 1c.

(2) (a) Evaluate the line integral

$$I = \int_C (2y\hat{x} + 3x\hat{y}) \cdot d\vec{\ell}$$

where the path C consists of straight line segments from $(0,0)$ to $(1,0)$ to $(0,2)$ to $(0,0)$. (b) Repeat, using Stokes' theorem. (5 marks)

(3) (a) Find the surface area of the portion of a plane $x + y + z = 1$ that lies within a cylinder $x^2 + y^2 = a^2$. (b) Is it possible to use the divergence theorem to solve this problem? (5 marks)

(4) Apply the separation of variables technique to solve the partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} - 2u$$

where $u(x, 0) = 10e^{-x} - 6e^{-4x}$. (5 marks)

(5) A half-range Fourier sine series is developed for $f(x) = x$ on $0 \le x \le 2$. (a) Sketch the Fourier series for $-6 \le x \le 6$. (b) Obtain the Fourier series. (5 marks)

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(1) Using arrows of proper magnitude and direction, sketch the vector function

$$\vec{A}(x,y) = \hat{x}y + \hat{y}xy$$

for $y = 1$, $-2 \leq x \leq 2$. (5 marks)

(2) Write a formula for a vector function in two dimensions which is in the positive radial direction and whose magnitude is 1. Give the final result in rectangular coordinates. (5 marks)

(3) Evaluate the line integral

$$I = \int_C (2y\hat{x} + 3x\hat{y}) \cdot d\vec{\ell}$$

where C is along the closed circular path $x^2 + y^2 = 1$ in the counterclockwise direction. Hints: $1 + \cos 2\phi = 2\cos^2\phi$, $1 - \cos 2\phi = 2\sin^2\phi$. (10 marks)

(4) Evaluate the surface integral

$$I = \int_S (x\hat{x} + y\hat{y} + 2z\hat{z}) \cdot \hat{n} dS$$

where the closed surface $S = S_1 + S_2$ is defined by a cone $S_1 : z = \sqrt{x^2 + y^2}; x^2 + y^2 \leq 1$ and a disk $S_2 : z = 1; x^2 + y^2 \leq 1$. Evaluate the integral directly, or use the divergence theorem. (10 marks)

(5) Apply the separation of variables technique to solve the partial differential equation

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$$

where $u(0,t) = 0$, $\partial u(x,t)/\partial x|_{x=2} = 0$, and $u(x,0) = 8\sin(3\pi x/4) - 6\sin(9\pi x/4)$. (10 marks)

Name: ....

Concordia University

Department of Electrical and Computer Engineering

EMAT 332 Vector Calculus and Partial Differential Equations

Midterm Exam: ***October 5, 2001***
Examiner: ***Mary Danial*** **Time Allowed:** ***1 Hour***

Question 1 (6 marks)

1.a. Sketch the vector field function using the arrows of proper magnitude and direction:

$$(iy + Jx)\Big/\sqrt{x^2 + y^2}$$

Convert to the cylindrical coordinates.

b. Determine whether or not $\int_C F.dr$ is independent of the path. If it is, find the potential function ψ

$$F(x,y) = (3x^2y + 2)\, i + (x^3 + 4y^3)\, j$$

Question 2 (8 marks)

Let S be the part of the graph of $z = 9 - x^2 - y^2$ where $z \geqslant 0$ and let C be the trace of S on the *xy-plane.* Verify stokes theorem.

Question 3 (8 marks)

Evaluate $\iint_S x^2 z \, d\, s$ where S is the portion of the cone $z^2 = x^2 + y^2$ which lies between the planes $z = 1$ and $z = 4$

Question 4 (8 marks)

Apply the separation of variables technique solve the partial differential equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad u_x(0,t) = 0, \quad u(2,t) = 0, \quad u(x,0) = 8\cos\frac{3\pi x}{4} - 6\cos\frac{9\pi x}{4}$$

good luck