1. (a) Given $f(z)=\dfrac{\left(z+i\sqrt{3}\right)^8}{(z-i)^4\left(3z+i\sqrt{3}\right)^6}$, find the value of f(1).

(b) Given $f(z)=z^{1/4}$, find all the values of $f\left(-2\sqrt{3}-2i\right)$. Sketch these values in the complex plane.

(12 marks)

2. Find all the solutions of cos z = 5.

(8 marks)

3. Map the region bounded by x = 1, y = 1, and x + y = 1 by the transformation $w = z^2$. Show clearly the region in the z-plane, and the corresponding image in the w-plane.

(10 marks)

4. Determine whether the function $u(xy)=x^2-y^2+4x-y$ is a harmonic function. If so, find the corresponding analytic function.

(8 marks)

5. If $\bar{z}$ denotes the complex conjugate of z, evaluate $\int \bar{z}\,dz$ from z = 0 to z = 1 + i

(a) along the line from z = 0 to z = 1 + i, and
(b) along the line from z = 0 to z = 1 first and then from z = 1 to z = 1 + i.
(c) Why is it that the integrals evaluated along the above two paths are different?

(9 marks)

6. Evaluate

$$\oint_C \frac{z^4}{(z-1-2i)^3}\,dz$$

C being taken in the counter-clockwise direction,

(a) when C is the quadrilateral C_1 whose vertices are at $z_a=0,\ z_b=3-i,\ z_c=3+3i,\ z_d=2i$

(b) when C is the circle C_2 whose centre is at the point z = 1 – 2i and is of radius 2

(c) when C is the circle C_3 whose centre is at the point z = 1 + i and is of radius 2

Give reasons for your answers.

(11 marks)

7. (a) Find the centre and radius of convergence of the power series

$$\sum_0^\infty \frac{(3+4i)^{n+1}}{(n+1)^3}\frac{1}{4^n}(z-i)^{2n}$$

(b) Determine if the following series is convergent or divergent.

(i) $$\sum_0^\infty \frac{(3+7i)^n}{11^n}$$

(ii) $$\sum_0^\infty \frac{2n+3i}{n-i}$$

(iii) $$\sum_0^\infty \frac{(10+3i)^{8n}}{(2n)!}$$

(10 marks)

8. Find the Taylor's series for the function $f(z)=e^z+\frac{1}{z}$ around the point z = 1. Also, determine its radius of convergence.

(8 marks)

9. Given $f(z)=\frac{1}{(z+1)(z+3)}$.

(a) It is intended to expand f(z) around z = 0. Determine the various regions over which Laurent series will be possible. (Do not obtain the series.)

(b) Is it possible to obtain a Taylor's series expansion for f(z) around the point z = 0? If so, determine the region over which it is possible. (Do not obtain the series.)

(c) Expand f(z) in Laurent series on $\frac{1}{2} < |z+1| < 2$. Find $\text{Res}_{-1} f(z)$.