Concordia University
Department of Mathematics and Statistics

Course	Number	Section	
EMAT	233/2	P	
Examination	**Date**	**Time**	**Pages**
Final	December 2001	3 hours	2
Instructor(s)			**Course Examiner**
A.Keviczky			R. Stern
Special Instructions	Answer all questions. NO CALCULATORS ALLOWED.		

Marks

[10] 1. Find the tangential and normal components of the acceleration at any time t for the position vector $\vec{r}(t)$ of a moving particle given by $\vec{r}(t) = (4\cos t, 3\sin t, 5t)$.

[10] 2. Given $F(x, y, z) = x^2 z^2 \sin(4y)$ find:

a) $(\nabla F)(-2, \frac{\pi}{3}, 1)$.

b) $\frac{\partial F}{\partial \vec{n}}$ at point $(-2, \frac{\pi}{3}, 1)$ in direction toward $(-\frac{5}{2}, \frac{\pi}{3}, 1 + \frac{\sqrt{2}}{2})$.

c) What are the extremal values of the directional derivative $\frac{\partial F}{\partial \vec{n}}$ at the point $(-2, \frac{\pi}{3}, 1)$?

[10] 3. For the given function $f(x, y) = 2x^2 y$ we let $x = x(u, v) = 2u + v^2$ and $y = y(u, v) = e^{u-v}$.

a) What is the composite function $F(u, v) = f\Big(x(u, v), y(u, v)\Big)$ in terms of (u, v).

b) Use the "Chain-rule" to obtain $\nabla F = (F_u, F_v)$. Your answer must be given strictly in terms of u and v.

[10] 4. Consider the line integral $\int\limits_C \vec{f} \cdot d\vec{r}$ for the vector field $\vec{f}(x, y, z) = \Big(6xz, -6yz, 3(x^2 - y^2)\Big)$.

a) Is $\int\limits_C \vec{f} \cdot d\vec{r}$ independent of the path? Justify your reply!

b) Evaluate $\int\limits_C \vec{f} \cdot d\vec{r}$ for $C : \vec{r}(t) = (3\cos t, 5, -2\sin t)(t : 0 \to 2\pi)$.

[10] 5. Let the double integral be given as the iterated integral $\iint\limits_D f(x, y)dA = \int_{x=0}^{1} \int_{y=x}^{1} f(x, y)dy\, dx$.

a) Describe the domain of integration D either as a set or by a drawing.

b) Reverse the order of integration.

MIDETRM EXAMINATION

EMAT 233/PRASAD

Time: 1 hour. Maximum points: 25 (5 points for each problem).

- Calculators are not permitted on this test.
- Do not evaluate expressions such as $\sqrt{3+4\pi^2}$. Just leave them as they are if they are a part of your answer.
- Please show all the steps in your solution and circle the final answer.

(1) Find the curl and divergence of the vector field

$$10yz\mathbf{i} + 2x^2z\mathbf{j} + 6x^3\mathbf{k}.$$

(2) Evaluate

$$\oint_C (x^2 - y^2)\,ds$$

where $x = 5\cos t$, $y = 5\sin t$, $0 \le t \le 2\pi$.

(3) Find the work done by the force field

$$\mathbf{F}(x, y) = 2x\mathbf{i} + 4y\mathbf{j}$$

along the curve

$$\mathbf{r}(t) = t\mathbf{i} + t^4\mathbf{j}, \quad 0 \le t \le 1.$$

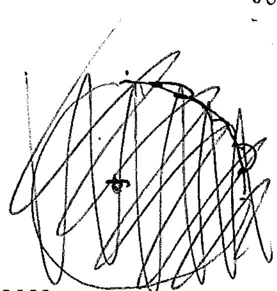
(4) Evaluate the integral

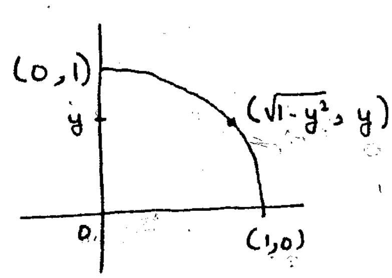
$$\int\int_R e^{x+3y}\,dA$$

over the region bounded by the graphs of $y = 1$, $y = 2$, $y = x$ and $y = -x + 5$.

(5) Evaluate by changing to polar coordinates (see figure)

$$\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2}\,dx\,dy.$$





$\int_0^1 \int_0^1 e^{x^2+y^2} dx\,dy$

Type 2

$x = \sqrt{1-y^2}$

$y = r \sin$

$x = r \cos$

Date: March 25, 2002.

C

EMAT 233/4 Section T

.32

TEST 2

Time allowed: 1 hour

No calculators.

March 27, 2002

MARKS

(10) 1. Find the work done by the force $\mathbf{F}(x, y, z) = (-y,\ x,\ -z)$ moving a particle along the helix $\mathbf{r}(t) = (\cos t,\ \sin t,\ 3t)$ for $0 \le t \le 2\pi$.

(10) 2. Compute the line integral

$$\int_C \cos x\, dx + z\, dy + y\, dz$$

where C is any curve starting at $(0, -1, -1)$ and ending at $(\pi, 3, 2)$.

(10) 3. Find the area of the region bounded by the parabola $y = x^2$ and the straight line $y = x + 2$.

(10) 4. Use Green's Theorem to evaluate

$$\oint_C (e^x + 2xy)\, dx + (e^y + 3x)\, dy$$

counterclockwise along the curve C: $x^2 + y^2 = 4$.

Concordia University

Final Examination - EMAT 233 - All sections

32

Date: April 2002
Time Allowed: 3 hours
Instructors: M. Babich, C. David, J. Hayes, A. Keviczky, Y. Khidirov, A. Pal, A. Prasad, R. Stern, A. Tupan
Course Examiner: C. David
Directions: Answer all questions. NO CALCULATORS.

MARKS

(12) **1.** Let C be the circular helix $\mathbf{r}(t) = 3\cos 2t\,\mathbf{i} + 3\sin 2t\,\mathbf{j} + t\,\mathbf{k}$.

(a) Find the equation of the tangent line at $P = (0, 3, \frac{\pi}{4})$.

(b) Find the curvature $\kappa(t)$.
Recall:

$$\kappa(t) = \frac{|\,\mathbf{r}'(t) \times \mathbf{r}''(t)\,|}{|\,\mathbf{r}'(t)\,|^3}.$$

(8) **2.** If $z = \ln(x^2 + y^2)$ with $x = x(r, s) = r^2 - s^2$ and $y = y(r, s) = r^2 s^2$, find

$$\frac{\partial z}{\partial r} \text{ and } \frac{\partial z}{\partial s}.$$

(8) **3.** Find the directional derivative of the function $f(x, y) = \sin\left(\frac{\pi x}{y}\right)$ at $P = (1, 4)$ and in the direction of the vector $\mathbf{v} = \mathbf{i} + \mathbf{j}$.

(8) **4.** Find the tangent plane to the surface $z = x^2 y$ at the point $P = (3, 2, 18)$.

(8) **5.** Let $\mathbf{F}(x, y, z) = (P(x, y, z), Q(x, y, z), R(x, y, z))$ be a sufficiently differentiable vector field. Show that $\operatorname{div}(\operatorname{curl} \mathbf{F}) = 0$.

(8) **6.** Find the work done by the force $F(x, y, z) = (x + yz, y^2, xy)$ acting along the curve given by $\mathbf{r}(t) = (t^3, t^2, t)$ for $0 \leq t \leq 1$.

(8) **7.** Consider the vector field $\mathbf{F}(x, y, z) = (6x^2, 4yz, 2y^2)$.

[10] 6. Evaluate $\iint\limits_R e^{4(x^2+y^2)} dA$ using polar coordinates $x = \rho\cos\theta$ and $y = \rho\sin\theta$, where $R\{(x, y) : x^2 + y^2 \leq 36$ and $x \leq 0, y \geq 0\}$.

[10] 7. Calculate the line integral $\oint\limits_C (x - 3y)dx + (4x - y)\, dy$ by the means of Green's theorem in the plane, where C is the circle $(x - 17)^2 + (y - 105)^2 = 4$ taken in the counter-clockwise direction.

[10] 8. Find the surface area of the portion of the paraboloid $z = x^2 + y^2$ that is below the plane $z = 9$.

[10] 9. For the vector field $\vec{f} = \vec{f}(x, y, z) = (5y, -5x, 3)$ verify Stokes' theorem for the surface $x + y + z = 6$ in the first octant oriented upward, i.e. $\oint\limits_{\partial S} \vec{f} \cdot d\vec{r} = \iint\limits_S (\nabla \times \vec{f}) \cdot d\vec{S}, d\vec{S} = \vec{n}dS$.

[10] 10. Use the Divergence Theorem to calculate the surface integral $\iint\limits_S \vec{f} \cdot d\vec{S} = \vec{n}dS$, for vector field $\vec{f} = \vec{f}(x, y, z) = (z + x^3, xz + y^3, e^{xy} + z^3)$ and $S = \partial R$ with outward orientation, where R is the solid $R = \{(x, y, z) : x^2 + y^2 + z^2 = 81, z \geq 0\}$.

(a) Find a scalar valued function $f(x, y, z)$ such that $\nabla f = \mathbf{F}$.

(b) Use the result of (a) to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is any path connecting the points $(1, 2, 3)$ and $(0, 1, 1)$.

(8) **8.** Find the volume of the region bounded above by the paraboloid $z = 1 + x^2 + y^2$, below by the plane $z = 0$ and inside the cylinder $x^2 + y^2 = 4$.

(8) **9.** Use Green's Theorem to evaluate

$$\int_C \left(7y + cos(x^2)\right)\, dx + \left(3x^2y - e^y\right)\, dy$$

along the curve C which is the boundary of the rectangle with vertices $(0, 0)$, $(2, 0)$, $(2, 1)$ and $(0, 1)$ with counterclockwise orientation.

(8) **10.** Find the surface integral

$$\iint_S xy\, dS,$$

where S is the portion of the surface $z = x^2 + 3y$ lying above the rectangle with vertices $(0, 0), (2, 0), (2, 1)$ and $(0, 1)$.

(8) **11.** Use Stokes' Theorem to evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where $\mathbf{F}(x, y, z) = (y^2, 3, 2x)$ and C is the circle $x^2 + z^2 = 1$ in the plane $y = 4$ with counterclockwise orientation as seen from the origin.

(8) **12.** Use the Divergence Theorem to evaluate the surface integral

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$$

where $\mathbf{F}(x, y, x) = (xy, yz, zx)$ and S is the unit cube $0 \leq x \leq 1$, $0 \leq y \leq 1$, $0 \leq z \leq 1$ parametrised with outward normal.

32

EMAT 233/4 Section T

TEST 1
Time allowed: 1 hour
No calculators.

February 15, 2002

MARKS

(8) 1. Consider the curve C given by the parametrisation $\mathbf{r}(t) = (e^t \cos t,\ e^t \sin t,\ e^t)$, $t \geq 0$.

(a) Graph the curve in $\mathbb{R}^3$.

(b) Find the tangent vector and the equation of the tangent line at the point $t = 2\pi$.

(8) 2. Compute the tangential and normal components of acceleration for $\mathbf{r}(t) = (t^2, -t^3, t^4)$.
Recall:

$$a_T = \frac{\mathbf{v}(t) \cdot \mathbf{a}(t)}{|\mathbf{v}(t)|}$$
$$a_N = \frac{|\mathbf{v}(t) \times \mathbf{a}(t)|}{|\mathbf{v}(t)|}$$

(10) 3. (a) Find the directional derivative of $w = x^2y^2z^3$ at the point $P_0 = (1, -1, 1)$ and in the direction of the vector $\mathbf{v} = (1, 2, 3)$.

(b) Find the equation of the tangent plane of the surface $x^2y^2z^3 = 1$ at the point $P_0 = (1, -1, 1)$.

(8) 4. Let $z = f(x, y)$ and $x = x(r, \theta) = r\cos\theta$, $y = y(r, \theta) = r\sin\theta$. Show that

$$\left(\frac{\partial f}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial f}{\partial \theta}\right)^2 = \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2.$$

[10] 6. Evaluate $\iint\limits_R e^{4(x^2+y^2)}dA$ using polar coordinates $x = \rho\cos\theta$ and $y = \rho\sin\theta$, where $R\{(x, y) : x^2 + y \le 36 \text{ and } x \le 0, y \ge 0\}$.

[10] 7. Calculate the line integral $\oint\limits_C (x - 3y)dx + (4x - y)\, dy$ by the means of Green's theorem in the plane, where C is the circle $(x - 17)^2 + (y - 105)^2 = 4$ taken in the counter-clockwise direction.

[10] 8. Find the surface area of the portion of the paraboloid $z = x^2 + y^2$ that is below the plane $z = 9$.

[10] 9. For the vector field $\bar{f} = \bar{f}(x, y, z) = (5y, -5x, 3)$ verify Stokes' theorem for the surface $x + y + z = 6$ in the first octant oriented upward. i.e. $\oint\limits_{\partial S} \bar{f} \cdot d\bar{r} = \iint\limits_S (\nabla \times \bar{f}) \cdot d\bar{S}.\, d\bar{S} = \bar{n}\, dS$.

[10] 10. Use the Divergence Theorem to calculate the surface integral $\iint\limits_S \bar{f} \cdot d\bar{S}.\, d\bar{S} = \bar{n}\, dS$. for vector field $\bar{f} = \bar{f}(x, y, z) = (z + x^3, xz + y^3, e^{xy} - z^3)$ and $S = \partial R$ with outward orientation, where R is the solid $R = \{(x, y, z) : x^2 + y^2 + z^2 = 81, z \ge 0\}$.