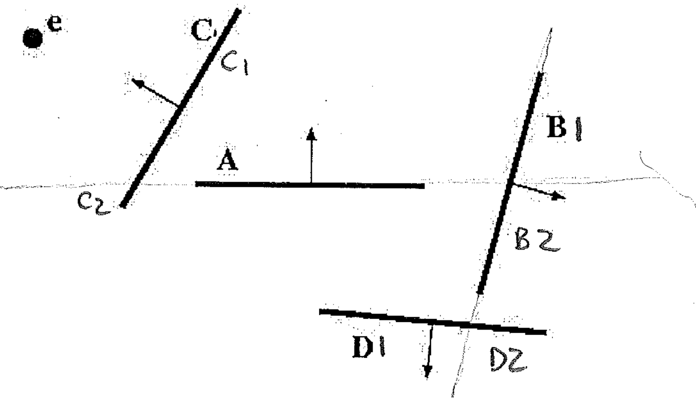
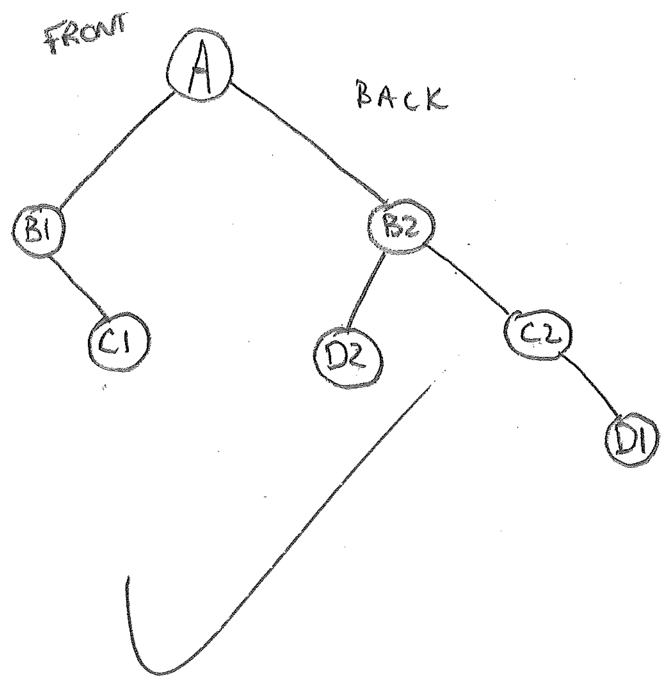
Question #1 [15 points ; 5 points each]

a) Create a BSP tree for the given scene (assume you are looking at a cross-section of the scene, including faces and their outward normals) using the algorithm presented in the lecture notes. Process the faces in an order such that if, at any time, there is a choice of several faces or face fragments, the one with the smallest alphabetical label is always chosen. Indicate in the scene the labels of any face fragments that are created while building your BSP tree. Clearly label the meaning of the left (right) subtree of a node in your BSP tree (just do so for the root node).



BSP tree:



Question #1 [continued.]

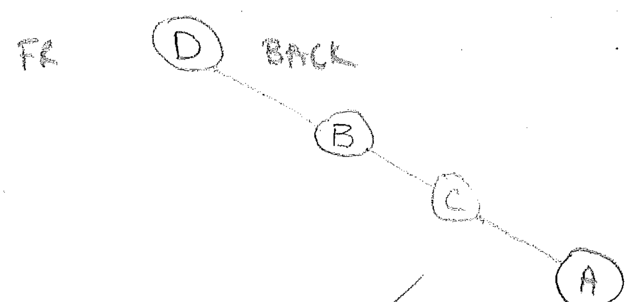
b) Given the BSP you just created in a), show the order in which to correctly draw the faces (or face fragments) using the Painter's algorithm given the eye position **e** shown above.

D2 B2 C2 D1 A B1 C1

4/5

c) Suppose that you could create your BSP tree by choosing the faces (or face fragments) in any order you wanted. What is the fewest number of nodes that you could have in your BSP tree? Give an example of such BSP tree for the same scene (hint : the tree will be different than that obtained in part a)).

4



COMP471, Quiz 2

Question #2 [25 points]

$R = 2(N \cdot L)N - L$

We use the Phong illumination model:

$$I = I_a k_a + I_p\left[k_d(\mathbf{N}\cdot\mathbf{L}) + k_s(\mathbf{R}\cdot\mathbf{V})^{n_s}\right]$$

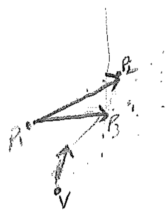
to determine how to shade a triangle T with vertices

$p_1 = (0, 1, 5)$, $p_2 = (1, 2, 1)$, and $p_3 = (2, 3, 3)$.

The viewer is located at $(0, -2, 3)$. We assume that $k_a = 0.3$, $k_d = 0.44$, $k_s = 0.17$, $I_a = 0.2$, $n_s = 2$, and the point light source is located at (2, 2, 2) with intensity $I_p = 0.4$

a) [12 points] Calculate the intensity I at the centroid of the triangle T.

Centroid $= \frac{A+B+C}{3} = \frac{3,6,9}{3} = (1,2,3)$ ✓



$V_1: P_2 - P_1 = (1, 1, -4)$
$V_2: P_3 - P_1 = (2, 2, -2)$

Light vector
$\vec{L} = l_{src} - \text{centroid}$
$= (1, 0, ~~-1~~)$

View Vector:
$\vec{V} = \text{viewer} - \text{centroid}$
$(-1, -4, 0)$

↑ normalize ↗ -2

$(V_2 \times V_1) \vec{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -2 \\ 1 & 1 & -4 \end{vmatrix} = (-6, 2, -1)$

$\|N\| = \sqrt{36+4+1}$ $\sqrt{40}$ $2\sqrt{10}$

$N \cdot L = \left(-\frac{3}{\sqrt{10}}, 0, \frac{1}{2\sqrt{10}}\right) = -\frac{5}{2\sqrt{10}}$

normalized: $\vec{n} = ~~\left(-\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}, -\frac{1}{2\sqrt{10}}\right)~~$ -0.5

$\frac{1}{\sqrt{2}}(1, -1, 0)$

reflection $=$ $R = 2(N \cdot L)N - L$

$= \left(-\frac{5}{\sqrt{10}} \cdot N\right) - L$

$= \left(\frac{18}{10}, -\frac{5}{10}, \frac{5}{20}\right) - L$

$R = \left(\frac{8}{10}, -\frac{5}{10}, \frac{25}{20}\right)$ -0.5

$R \cdot V = \left(-\frac{8}{10} + \frac{20}{10} + 0\right) = \frac{12}{10}$

$I = I_a k_a + I_p\left[k_d(N \cdot L) + k_s(R \cdot V)^{n_s}\right]$

$I = (0.2)(0.3) + 0.4\left[(0.44)\left(-\frac{5}{2\sqrt{10}}\right) + 0.17\left(\frac{6}{5}\right)^2\right]$ ← hard to calculate since we're not allowed hand calculators.

-1

$8/12$

$V_1 0$ $V_1 1$ $V_1 2$
$V_2 0$ $V_2 1$ $V_2 2$
$(V_1 0 V_2 2 - V_1 2 V_2 0)$

Question #2 [continued.]

b) [4 points] Write an OpenGL function TNormal() that computes the normal of the triangle.

```
void TNormal( GLfloat p1[], GLfloat p2[], GLfloat p3[], GLfloat N[]) {
  GLfloat V1[] = new float[]
  GLfloat V2[] = new float[]
  for (int i=0 ; i<3 ; i++)
   {
      V1[i] = p2[i] - p1[i];
      V2[i] = p3[i] - p1[i];
   }

  N[0] = V1[1]*V2[2] - V1[2]*V2[1];
  N[1] = V1[2]*V2[0] - V1[0]*V2[2];
  N[2] = V1[0]*V2[1] - V2[0]*V1[1];
float sumFac = sqrt(N[0]*N[0] + N[1]*N[1] + N[2]*N[2])
    N[0] = N[0]/sumFac ;  N[1] = N[1]/sumFac ;  N[2] = N[2]/sumFac.

} // Tnormal
```

4/4

c) [2 points] Give one advantage and one disadvantage of Phong shading compared to Gouraud shading?

disadvantage: It takes much longer to compute because the Normal are interpolated as oppose to vertex intensity.

advantage: It is much more realistic. Will show Specular reflections correctly.

2/2

Question #2 [continued.]

d) [5 points] Find the implicit and parametric equations of the plane defined by the triangle T.

$ax + by + cz - d = 0$

0

e) [2 points] Show the equations in f) are consistent.

TIMES UP!

COMP471, Quiz 2

Question #3 [20 points]

$(z-1)(z-1)$
$(z+1)(z^2-2z+1)$
$z^3-2z^2+z+z^2-2z+1$
z^3-z^2-z+1

A surface is defined implicitly by the function f given by

$$f(x,y,z)=\frac{x^3}{3}+\frac{y^2}{2}+(z+1)(z-1)^2=0.$$

We assume Phong illumination model given by:

$$I=I_a k_a+I_p\left[k_d(\mathbf{N}\cdot\mathbf{L})+k_s(\mathbf{R}\cdot\mathbf{V})^{n_s}\right]$$

where $\mathbf{L}=(0,1/\sqrt{2},1/\sqrt{2})$ and $\mathbf{V}=(1/\sqrt{2},0,-1/\sqrt{2})$.

a) [8 points] Calculate the unit normal vector $\mathbf{N}=\nabla f/\|\nabla f\|$ at the point p = (0, 3, 1).

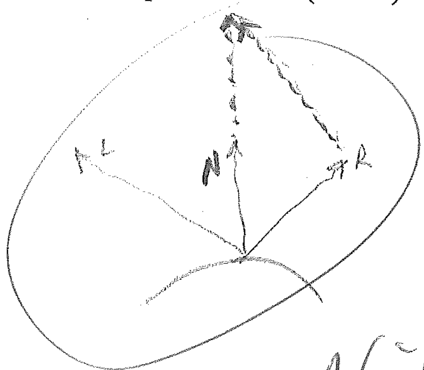
$\nabla f = (x^2,\ y,\ 3z^2-2z-1)$
$\nabla f(0,3,1) = (0,3,0)$
$N = \frac{\nabla f}{\|\nabla f\|} = (0,1,0)$

b) [2 points] Calculate the value of $\mathbf{N}\cdot\mathbf{L}$ at the point p = (0, 3, 1).

$N\cdot L = (0\cdot 0 + 1\cdot 1/\sqrt{2} + 0\cdot 1/\sqrt{2})$
$N\cdot L = 1/\sqrt{2}$

Question #3 [continued.]

c) [8 points] Show ~~geometrically (aided by a diagram) that the reflectance vector is~~ equal to **R** = 2(**N** · **L**)**N** − **L**, and calculate the value of **R** at the point p = (0, 3, 1).



$R = 2(1/\sqrt{2})N - L$

$= 2/\sqrt{2}\,N - L$

$= (0, 2/\sqrt{2}, 0) - L$

$R = (0, 1/\sqrt{2}, -1/\sqrt{2})$

more detail

6/8

d) [2 points] Calculate the value of **R** · **V** at the point p = (0, 3, 1).

$R \cdot V = (0 \cdot 1/\sqrt{2} + 1/\sqrt{2} \cdot 0 + -1/\sqrt{2} \cdot -1/\sqrt{2})$

$= 1/2$

Question #4 [40 points ; 5 points each]

Description: *Each question has four possible choices of which* ***ONLY ONE*** *is the correct answer. Read the text of the question carefully, then read ALL the choices and tick ONLY one of the four choices as your choice for the CORRECT answer. Ticking more than one choice for a single question will normally be considered as an INCORRECT answer. Show your working of the answer on the side of the question or on the blank side of the question paper. In case, you find some question for which you think more than one answer is correct, then please give the explanation for it and choose any one.*

1. Consider the circle defined by equation $(x - 2)^2 + (y + 1)^2 = 25$. What can you say about the relationship between this circle and the points (6,2) and (2,4)?

   (A) They are both inside the circle.
   (B) They are both on the circle.
   (C) They are both outside the circle.
   (D) One is inside, the other is outside.

   16 + 9 = 25 = 25
   0 + 25 = 25 = 25

2. A scene consists of 4 planar faces lying on planes given by the following 4 equations:

   $-x - 3y + 2z = 0$ $\frac{-1, -3, 2}{\sqrt{14}}$
   $x + 2y + 4z = 0$ $\frac{1, 2, 4}{\sqrt{21}}$
   $4x - 3y + z + 8 = 0$ $\frac{4, -3, 1}{\sqrt{26}}$
   $3x + 2y + z - 6 = 0$ $\frac{3, 2, 1}{\sqrt{14}}$

   find dot product
   $n \cdot \vec{V} > 0$ then remove.
   So none since all $\vec{n}$ point to +z

   Before rendering, all the back faces are removed. If this scene was to be rendered from a distant point as viewed along the directed line with (0,0,-1) being the view direction then how many of these 4 faces would be eliminated?

   (A) 0 (B) 1 (C) 2 (D) 4

3. Consider a scene with three triangles with the following coordinates

   1 T_1 – (0, 0, 1), (5, 0, 0), (0, 5, 0)
   3 T_2 – (2, 5, 7), (7, 5, 5), (2, 10, 6)
   2 T_3 – (5, 0, 2), (10, 0, 3), (5, 5, 4)

   25/40

   If the scene is being rendered as viewed from the point (0, 0, 50) in the negative z-direction using Painter's Algorithm, what is the order in which the triangles would be rendered?

   (A) T_1, T_2, T_3
   (B) T_3, T_2, T_1
   (C) T_2, T_3, T_1
   (D) T_1, T_3, T_2

4. Consider the following intensity model for reflection from a surface in a system with ambient light and a single point-light-source taking into consideration both the diffused as well as specular components of reflection.

$$I = I_a k_a + \sum_i I_i \left(k_d (cos\theta) + k_s (cos\phi)^{n_s} \right)$$

From the following four statements made about this equation, pick out the **FALSE** one.

(A) k_d is high for weakly reflective surfaces.

(B) ✓ For computing $(cos\theta)$ we need the surface normal vector.

(C) For perfect mirror effects the value of "n_s" should approach a very large value. R should be in direction of View Vector.

(D) $I_a k_a$ represents the component due to illumination by light reflected by other surfaces. Ambient reflection .....

5. In Gouraud shading, the colour at any point lying inside the polygon is computed by,

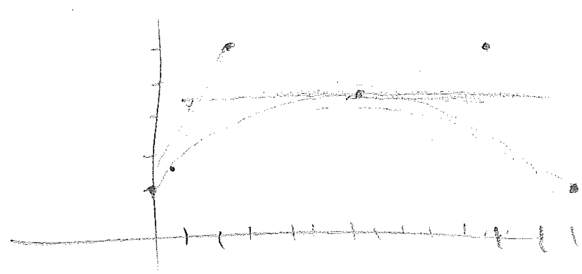
(A) average of the colour at the vertices of the polygon.
(B) interpolating the normals at the vertices of the polygon.
(C) interpolating the colour at the vertices of the polygon.
(D) equating it to the colour of the vertex.

6. A cubic Bézier curve P is defined by the control points as shown below :

$P_0 = (0,1)$, $P_1 = (2,5)$, $P_2 = (14,5)$, $P_3 = (16,1)$

Choose the most likely tangent direction for the point at u=0.5 on curve P.

(A) (1,0)
(B) (1,1)
(C) (2,-4)
(D) (2,4)



7. The following are the control points of two Bézier cubics which are tangent (C^1) continuous [(2,3), (3,8), (10,9), (12,5)], and [(12,5), P, (18,4), (20,3)]

   Which one of the following could be the coordinates of point P?

   (A) (13,1) (B) (14,3) (C) (14,1) (D) (13,2)

$\frac{5-9}{12-10} = \frac{-4}{2} = -2$

$\frac{y-5}{x-12} = -2$

$y-5 = -2x+24$

$2x+y = 29$

$2(14)+(1) = 29$

$28+1 = 29$

8. Which of the following is **INCORRECT**?

   (A) Radiosity is best suited for computing global illumination in completely diffuse environments.

   (B) Ray tracing technique is better suited for computing specular reflections.

   (C) Radiosity is not suitable to model refraction effects.

   (D) Ray tracing is not suitable to model refraction effects.

------------------------------------ **END OF QUESTION PAPER** ------------------------------------