.07

COMP 335 Theoretical Computer Science
Winter 1997 Mid-term 1

Time: 1 hour 15 minutes

Notes: (1) This exam will be graded out of 15 points.
(2) Only complete explanations will get full credit.

1. [2] State the pumping lemma for regular languages.
2. [3] Prove that the language $L_1 = \{a^i b^j a^j \mid i, j \geq 0\}$ is not regular.
3. [5] Prove that the following languages are regular:
   (a) $L_2 = \{w \in (a+b)^* \mid n_a(w)$ is odd or $n_b(w)$ is even $\}$
   (b) $L_3 = \{w \in (a+b)^* \mid |w| \geq 1$ and the last letter of w is the same as the first letter of $w\}$
4. Consider the language $L = \{w \in (a+b)^* \mid$ the string aba does not appear in $w\}$.
   (a) [2] Find a DFA that accepts L.
   (b) [1] Convert it into an equivalent right regular grammar.
   (c) [2] Find a regular expression that represents $\overline{L}$.

Bonus:

5. [1] Let $w = w_1 w_2 \dots w_n$ be a string in Σ^* where each w_i is a symbol in the alphabet Σ. Then $w^R = w_n w_{n-1} \dots w_2 w_1$. Given a regular expression for a language L, show how to construct a regular expression for $Reverse(L) = \{w \mid w^R \in L\}$.

Sample (Sol^n)

COMP 335 Theoretical Computer Science
Winter 1996 Mid-term 2

Time: 1 hour 15 minutes

1. [6] For each of the following languages, say whether or not it is context-free. Explain your answer.
   (a) $L_1 = \{uavb \mid u, v \in (a+b)^*, |u| = |v|\}$
   (b) $L_2 = \{a^n b^n c^k \mid n \leq k \leq 2n\}$

2. [3] Say true or false, providing a complete explanation for your answer. Answers without correct explanations will get NO credit.
   (a) If $L_1 \subseteq L_2$ and L_1 is not context-free, then L_2 is not a context-free language.
   (b) The grammar given below is unambiguous.
   $S \longrightarrow SS \mid aSb \mid bSa \mid \lambda$
   (c) The language $\{a^n b^{2^n} \mid n \geq 0\}$ is a deterministic context-free language.

3. [4] Consider the language $L = \{a^l b^n c^p d^q \mid l + n = p + q\}$
   (a) Give a CFG that generates the language.
   (b) Give a push down automaton that accepts the same language. You may either give a direct construction, or convert the context-free grammar you derived above to an equivalent PDA.

4. [2] Suppose L is a context-free language and R is a regular language. Is $L-R$ necessarily context-free? How about $R-L$? Provide explanations for your answers.

Bonus Question:
[2] Show that the conversion to Chomsky Normal Form can square the number of productions in a grammar.

Solutions for Midterm 2 COMP 335. k4 (1)

Winter 1996 .13

1. Say whether or not context-free.

(a) $L_1 = \{uavb \mid u, v \in (a+b)^*, \ |u| = |v|\}$

L_1 is context-free; it is generated by the following grammar

$S \to Ab$

$A \to XAX \mid a$

$X \to a \mid b$

(b) $L_2 = \{a^n b^n c^k \mid n \le k \le 2n\}$

L_2 is not cfree. Suppose it is. Then let m be the constant of the pumping lemma.

Choose $w = a^m b^m c^m$. $|w| \ge m$ & $w \in L$

Let $w = uvxyz$ with $|vxy| \le m$ and $|vy| \ge 1$

We consider all the possibilities for choices of v and y.

Case 1: $vy \in a^+$. Then $\#a(uxz) \ne \#b(uxz)$ $\therefore uxz \notin L$

Case 2 $vy \in b^+$. Then $\#a(uxz) \ne \#b(uxz)$ $\therefore uxz \notin L$

Case 3 $vy \in c^+$ Then $\#c(uxz) < \#a(uxz)$ $\therefore uxz \notin L$

Case 4 $vy \in a^+b^+$ Then $\#c(uv^2xy^2z) < \#a(uv^2xy^2z)$ $\therefore uv^2xy^2z \notin L$

Case 5 $vy \in b^+c^+$ Then $\#a(uxz) \ne \#b(uxz)$ $\therefore uxz \notin L$

Case 6 $vy \in a^+c^+$ Impossible since $|vxy| \le m$

Case 7 $vy \in a^+b^+c^+$ Impossible since $|vxy| \le m$.

* 2. Say true or false.

(a). If $L_1 \subseteq L_2$ and L_1 not cfree then L_2 is not cfree.

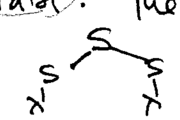
False. Consider $L_1 = \{a^n b^n c^n \mid n \ge 0\}$. L_1 is not cfree

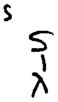
$L_2 = a^* b^* c^*$. L_2 is regular, $\therefore$ cfree.

But $L_1 \subseteq L_2$, and contradicts the given statement.

(b) The grammar $\{S \to SS \mid aSb \mid bSA \mid \lambda\}$ is unambiguous.

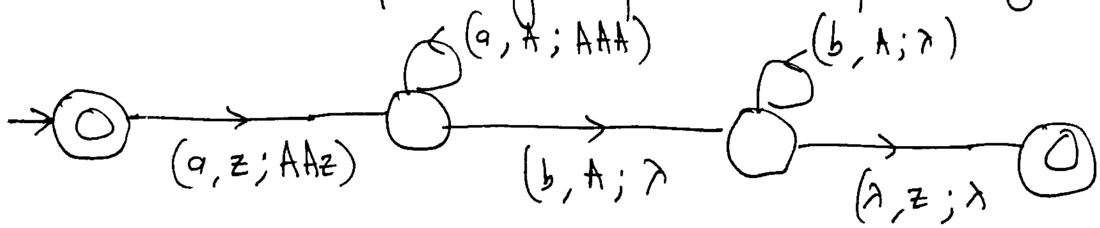
False. The string λ has more than one derivation.





(c) The language $\{a^n b^{2n} \mid n \geq 0\}$ is a dcfl. ②

True. The following dpda accepts the given language



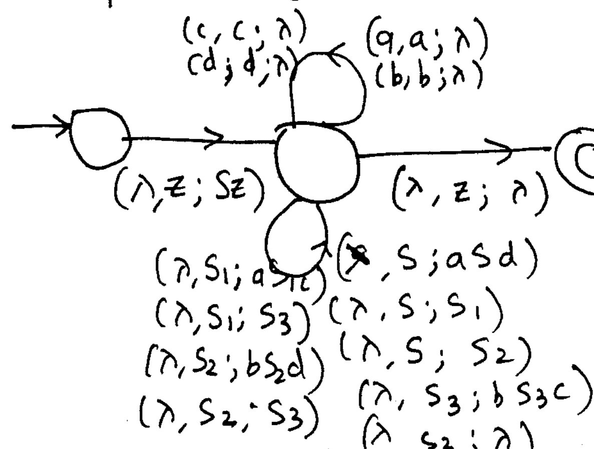
3. Give a cfg and pda to accept $L = \{a^l b^n c^p d^q \mid l+n = p+q\}$

$S \rightarrow aSd \mid S_1 \mid S_2$

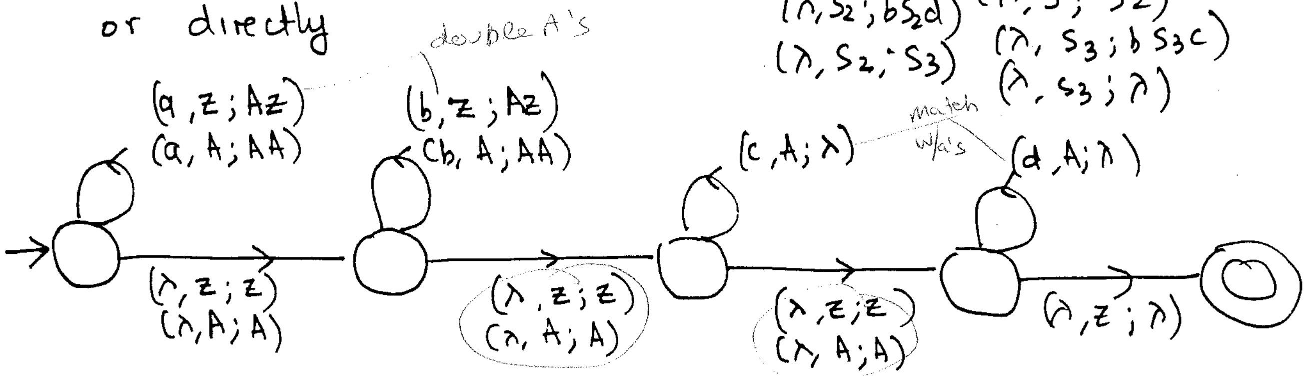
$S_1 \rightarrow aS_1c \mid S_3$

$S_2 \rightarrow bS_2d \mid S_3$

$S_3 \rightarrow bS_3c \mid \lambda$



or directly



4. Suppose L is cfree and R is regular. Is $L-R$ necessarily cfree? How about $R-L$?

$L-R = L \cap \bar{R}$. Since R is regular, $\bar{R}$ is regular (as regular reg languages are closed under complementation). Since L is cfree and $\bar{R}$ is regular $L \cap \bar{R}$ is cfree, since context-free languages are closed under intersection with reg. languages.

Suppose $R-L$ was always context-free. Then let $R = \Sigma^*$. $R-L = \bar{L}$. This implies that if L is context-free, $\bar{L}$ is context-free. But we know that cfree langs are not closed under complementation. Thus, we get contradiction, and ∴ our assumption that $R-L$ is always cfree is wrong.

∴ $R-L$ is not necessarily cfree.

Bonus Show that the conversion to CNF can square the # of productions in a grammar.

Ans Use the ~~co~~ removal of unit productions

1K

Sample

Soln

Jimmy

MIDTERM

Comp 335 - Sec. XX
Adam Steele

60 mins

Note : Unless stated otherwise, you can assume an FA may be incomplete. Good Luck.

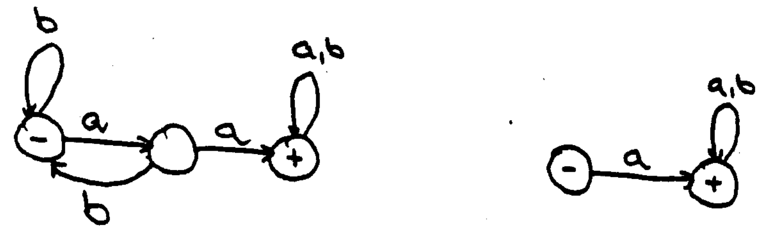
[25] 1. Find a TG for the regular expression (ab + aab + aba)*. Convert the TG to a complete FA.

[30] 2. Show, by formal proof, that the following equalities are true.

i) $(aa)^*(\Lambda + a) = a^*$

ii) $(b^*a^*)^* = (a + b)^*$

[30] 3. For the following two FA's.



Find an FA that describes the language that is the intersection of the languages described by the machines above. What is the regular expression that is equivalent to this new FA.

[15] 4. If L is a regular language prove that Pref(L) is also a regular language.

Pref(L) = {x : for some y, xy is in L}

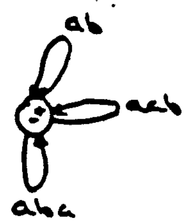
(Pref(L) is the set of prefixes of L, i.e. Λ, ab^*, a and ab^*a, are prefixes of ab^*a).

[1] ~~5. (Bonus)~~ : how many Brandenburg Concertos ~~are there?~~ 6.

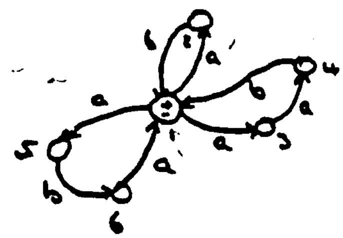
5-10

Adam Steck

1) TG representing $(ab+aab+aba)^*$



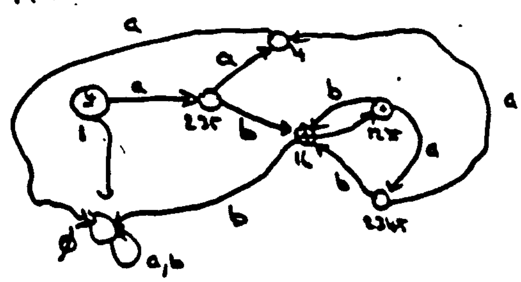
transforming to NFA ⇒



Removing non-determinism

$1a = 235$	$4a = \emptyset$	$1235a = 2345$
$1b = \emptyset$	$4b = 1$	$1235b = 16$
$235a = 4$	$16a = 1235$	$2345a = 4$
$235b = 16$	$16b = \emptyset$	$2345b = 16$

Complete FA:



2) i Proof of $(aa)^*(\Lambda+a) = a^*$

$$
\begin{aligned}
(aa)^*(\Lambda+a) &= (aa)^n(\Lambda+a) \\
&= a^{2n}(\Lambda+a) \\
&= a^{2n} + a^{2n+1} \quad \text{distributivity} \\
&= a^k \quad \text{any \# is either even or odd} \\
&= a^*
\end{aligned}
$$

even/odd

ii $(b^*a^*)^* = (a+b)^*$

Part 1: $(b^*a^*)^* \subseteq (a+b)^*$

Trivially true, (prove by contradiction)

2) ii contd. * Part 2: $(a+b)^* \subseteq (b^*a^*)^*$

$a \in b^*a^*, b \in b^*a^* \Rightarrow (a+b) \subseteq b^*a^*$
$(a+b)^* \subseteq (b^*a^*)^*$ property of sets $S \subseteq T \Rightarrow S^* \subseteq T^*$ (lemma 1)

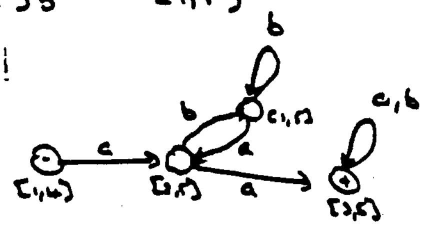
Or by induction
from Parts 1 and 2 $(b^*a^*)^* = (a+b)^*$

*3)

initial state = [1,4]

[1,4]a = [2,5]
[1,4]b = [1,∅] = ∅
[2,5]a = [3,5]
[2,5]b = [1,5]

[3,5]a = [3,5] } terminal state
[3,5]b = [3,5]
[1,5]a = [2,5]
[1,5]b = [1,5]



$\Rightarrow a(bb^*a)^*a(a+b)^*$

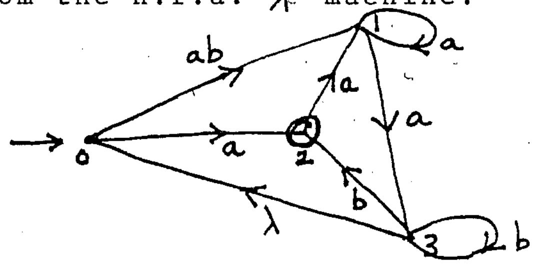
*4) If L is regular the L is accepted an FA, M = (Q,i,T), make every state a final state giving M' = (Q,i,Q). M' accepts Pref(L). Hence Pref(L) is regular.

~~5) J.S. Bach composed six Brandenburg concertos.~~

COMP335/2 Section U   Midterm   Oct. 26th. 1993   Time: 30 minutes

No notes, no books. Give full explanations for full marks.

1. From the n.f.a.-λ machine:



Construct an equivalent deterministic finite automaton. Give all steps in your construction.

2. (a) By drawing a graph of dependency among the non-terminals, or otherwise, show that the grammar given generates a finite language, and give a derivation of a longest word.

$S \rightarrow XY$, $X \rightarrow YZ \mid a$, $Y \rightarrow ZZ \mid b$, $Z \rightarrow a$

(b) Adding the production rule $Z \rightarrow XY$, show that the language is now infinite & give a regular expression for an infinite string in it.

3. Construct a p.d.a. from the grammar:

$S \rightarrow zXYz$, $X \rightarrow aXa \mid z$, $Y \rightarrow bYb \mid z$

....
Name Student Id

Concordia University
Department of Computer Science

COMP 335 – Introduction to Theoretical Computer Science

Section WW, Fall 1998

Mid-Term Test
November 24, 1998

The time allowed is 90 minutes. Total 20 marks. No materials are allowed.

1. Let $L = \{x \in \{a,b\}^* \mid n_a(x) < 3n_b(x)\}$. Show that L is nonregular.

   SOLUTION. Let $x = a^{3n-1}b^n$. Since $|uv| \le n$, $uv = a^k$ for some k, $0 < k \le n$. Since $|v| > 0$, $v = a^j$ for some j, $0 < j \le k$. Hence $n_a(uv^2w) = n_a(a^{3n-1+j}b^n) = 3n - 1 + j \ge 3n = 3n_b(a^{3n-1+j}b^n)$, so $uv^2w \notin L$.

2. Find a CFG generating the language $\{x \in \{a,b\}^* |$the two middle symbols of x are equal$\}$.

   SOLUTION. $S \rightarrow aa \mid bb \mid aSa \mid aSb \mid bSa \mid bSb.$

3. Find a CFG in Chomsky normal form generating the same language as the following grammar.

$$
\begin{array}{lcl}
S & \rightarrow & SABC \mid ab \mid SA \mid SAC \mid SAB \\
A & \rightarrow & Aab \mid Bb \mid b \\
B & \rightarrow & CDc \mid b \mid \Lambda c \mid Cc \mid Dc \\
C & \rightarrow & BD \mid abD \mid B \mid D \mid ab \\
D & \rightarrow & Dc \mid \Lambda c
\end{array}
$$

SOLUTION.

1. Eliminating Λ-productions.

$$
\begin{array}{lcl}
S & \rightarrow & SABC \mid SAB \mid SAC \mid SA \mid ab \\
A & \rightarrow & Aab \mid Bb \mid b \\
B & \rightarrow & CDc \mid Cc \mid Dc \mid c \mid b \\
C & \rightarrow & BD \mid B \mid D \mid abD \mid ab \\
D & \rightarrow & Dc \mid c
\end{array}
$$

2. Eliminating unit-productions.

$$
\begin{array}{lcl}
S & \rightarrow & SABC \mid SAB \mid SAC \mid SA \mid ab \\
A & \rightarrow & Aab \mid Bb \mid b \\
B & \rightarrow & CDc \mid Cc \mid Dc \mid c \mid b \\
C & \rightarrow & BD \mid CDc \mid Cc \mid Dc \mid c \mid b \mid abD \mid ab \\
D & \rightarrow & Dc \mid c
\end{array}
$$

3.

$$
\begin{aligned}
S &\rightarrow SABC \mid SAB \mid SAC \mid SA \mid EF \\
A &\rightarrow AEF \mid BF \mid b \\
B &\rightarrow CDG \mid CG \mid DG \mid c \mid b \\
C &\rightarrow BD \mid CDG \mid CG \mid DG \mid c \mid b \mid EFD \mid EF \\
D &\rightarrow DG \mid c \\
E &\rightarrow a \\
F &\rightarrow b \\
G &\rightarrow c
\end{aligned}
$$

4.

$$
\begin{aligned}
S &\rightarrow SY_1 \mid SY_3 \mid SY_4 \mid SA \mid EF \\
A &\rightarrow AY_5 \mid BF \mid b \\
B &\rightarrow CY_6 \mid CG \mid DG \mid c \mid b \\
C &\rightarrow BD \mid CY_6 \mid CG \mid DG \mid c \mid b \mid EY_7 \mid EF \\
D &\rightarrow DG \mid c \\
E &\rightarrow a \\
F &\rightarrow b \\
G &\rightarrow c \\
Y_1 &\rightarrow AY_2 \\
Y_2 &\rightarrow BC \\
Y_3 &\rightarrow AB \\
Y_4 &\rightarrow AC \\
Y_5 &\rightarrow EF \\
Y_6 &\rightarrow DG \\
Y_7 &\rightarrow FD
\end{aligned}
$$

4. Give a deterministic PDA recognizing the language generated by the following grammar.

$$
\begin{aligned}
S &\rightarrow S_1\$ \\
S_1 &\rightarrow aAbB \mid bB \\
A &\rightarrow bA \mid \Lambda \\
B &\rightarrow aB \mid b
\end{aligned}
$$

SOLUTION. Top-down parser:

State	Input	Stack symbol	Move(s)
q_0	Λ	Z_0	(q_1, SZ_0)
q_1	Λ	S	$(q_1, S_1\$)$
q_1	a	S_1	$(q_a, aAbB)$
q_1	b	S_1	(q_b, bB)
q_1	a	A	(q_a, Λ)
q_1	b	A	(q_b, bA)
q_1	$\$$	A	$(q_\$, \Lambda)$
q_1	a	B	(q_a, aB)
q_1	b	B	(q_b, b)
q_1	a	a	(q_1, Λ)
q_1	b	b	(q_1, Λ)
q_1	$\$$	$\$$	(q_1, Λ)
q_a	Λ	a	(q_1, Λ)
q_b	Λ	b	(q_1, Λ)
$q_\$$	Λ	$\$$	(q_1, Λ)
q_1	Λ	Z_0	(q_2, Z_0)

Midterm Examination 1

1. *(20%)* The reverse of a string can be defined by the recursive rules

$$a^R = a,$$

$$(wa)^R = aw^R,$$

for all $a \in \Sigma$, $w \in \Sigma^*$. Use this to prove that

$$(uv)^R = v^R u^R,$$

for all $u, v \in \Sigma^+$.

2. *(40%)* Let $L = L[(a+b)^*] - L[(bab)^*]$.
   i) Give a dfa that accepts L
   ii) Give a regular expression for L.

3. *(20%)* Let

$$M_1 = (Q, \Sigma, \delta_1, q_0, F_1)$$

$$M_2 = (P, \Sigma, \delta_2, p_0, F_2)$$

be nfa's, with $Q \cap P = \Phi$, construct an nfa M such that

$$L(M) = L(M_1) \cup L(M_2).$$

Prove the construction.

4. *(20%)* Show that the language

$$L = \{waw : w \in \{a, b\}^*\}$$

is not regular.