COMP 231/2 — Fall 1994
Midterm No. 1

by both ways

1. Prove or disprove whether $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \wedge q) \rightarrow r)$ is a tautology.

2. Assume

   the UoD of x is "persons", the predicate $S(x)$ denotes "x is a student",
   y is "stuff", $F(y)$ denotes "y is food",
   $E(x,y)$ denotes "x eats y",
   $W(y)$ denotes "y is white".

   Express the following in predicate logic:

   (a) Some students eat any food.
   (b) For every student there is at least one food that the student doesn't eat.
   (c) There is a student who eats only white food.

3. Show that $\forall x\ P(x) \vee \forall x\ Q(x)$ and $\forall x\ (P(x) \vee Q(x))$ are not logically equivalent.

4. Let $A = \{\{a\}, \{\{a\}, a\}, a\}$.
   Give the power set P(A).

5. Prove using set equality or disprove using a counterexample that for all sets A, B, and C it is true that

   (a) $A - (B - C) = (A - B) \cup (A \cap C)$
   (b) $A - (B \cup C) = (A - B) \cup (A - C)$

6. Show whether the following are bijections:
   (Hint: Remember that a bijection is a function that is both one-to-one and onto.)

   (a) $f : \mathbf{R} \longrightarrow \mathbf{R}$, where $f(x) = \frac{x^2+1}{x^2+2}$
   (b) $f : \mathbf{Z} \longrightarrow \mathbf{Z}$, where $f(x) = \lceil \frac{x}{2} \rceil$
   (Hint: $\lceil a \rceil$ is the function that maps any real number to the next larger integer, thus $\lceil a \rceil = b$, where $b - 1 < a \leq b$.)
   (c) $f : \mathbf{R} \longrightarrow \mathbf{R}$, where $f(x) = 2x + 1$.

COMP 231/4-03, WINTER 1993 CONCORDIA UNIVERSITY

TEST 1.

Instructor: A. Krzyzak.

Date: February 11, 1993.
Duration 75 minutes.

Total points: 100.
15% of final mark.

1. (20%) Is it true that $(p \longrightarrow q) \longrightarrow r$ is equivalent to $p \longrightarrow (q \longrightarrow r)$? Prove your answer using propositional laws.

2. (20%) Given two sets A and B, prove that $A \cup B \subseteq A \cap B$ implies $A = B$.

3. (20%) Write each of the following predicates in symbolic form using appropriate quantifiers.

   a) (10%) "N_0 is the least x for which $P(x)$ is true.

   b) (10%) "There is a unique x for which $P(x)$ is true".

4. (20%) Given any integer $n > 1$, prove by contradiction that the least integer > 1 which divides n must be prime. You must state the negation of the theorem.

5. (20%) Let $A = B = \mathcal{P}(S)$, where $\mathcal{P}(S)$ is a power set of set S. If $X \in \mathcal{P}(S)$, let $f : A \rightarrow B$ and $f(X) = \overline{X}$.

   a) Prove whether f is one-to-one and onto.

   b) Prove whether the inverse function f^{-1} exists. If the inverse exists find it.

6. (20%) (bonus) Given $f : A \rightarrow B$, prove the following.

   - f is one-to-one $\leftrightarrow \forall C \subseteq A\,(f \circ f^{-1}(C) = C)$

$f \quad f \circ f^{-1}(c) = c$
$C \subseteq A$

Concordia University

Department of Electrical and Computer Engineering

COEN 231/2 U Fall 1996
Discrete Mathematics

Final Examination

Value: 65% of final grade
Time: 3 hours
Total marks: 65

Materials allowed: Two 8 and 1/2 by 11-crib sheets (both sides)
Any inanimate computing aids you can manage to carry in.

Note: It is not necessary to work out factorials on your calculator. Doing so will not give you extra marks.

1a). A deck of 52 cards contains 4 aces, 4 kings, 4 queens 4 jacks, 4 tens,..... and 4 twos. A bridge hand is formed by selecting 13 cards at random.

How many bridge hands contain exactly 3 aces and 2 queens? (4 marks)

1b). Find the number of ways of distributing 8 coins among four containers where:
i) There are no restrictions. (2 marks)

ii) Container number four must have an odd number of coins. (4 marks)

2a). Using truth tables, prove the following distributive law of logic:
$(p \wedge q) \vee r \Leftrightarrow (p \vee r) \wedge (q \vee r)$. (5 marks)

2b). Using the laws of logic (not truth tables) prove that:
$p \rightarrow (p \wedge q) \Leftrightarrow p \rightarrow q$. (5 marks)

2c). Prove the following argument:

$(p \rightarrow q) \rightarrow (r \wedge t)$

$(r \wedge t) \rightarrow s$

$\underline{\neg s}$

$\therefore p$ (5 marks)

2d). Let $x \in$ **R.** We are given the open two statements $p(x)$: $x^2 \geq 4$ and $q(x)$: $x < 6$. Determine the values of x such that $p(x) \wedge q(x)$ is true. (5 marks)

3. Prove by induction that the solution to the difference equation:
$y_n - 0.5y_{n-1} = n, \; y_0 = 0$, is

$y_n = 2[(0.5)^n + n - 1]$.

(Do not prove by finding the solution - this is a question on induction.) (10 marks)

4a). Find the solution to $y_n - ay_{n-1} = b^n, \; y_0 = 1$ (8 marks)

4b). It is determined that the unit pulse response of a given linear time invariant system is $g_n = \{1,2,3,2,1,0,0..\}$.

i) For a given input $x_n = \{1,1,1,0,0,......\}$, find the ouput y_n. Plot the first ten terms of your result. (4 marks)

ii) Suppose the input is $x'_n = \{0,0,2,2,2,0,......\}$.
Use the results of part a) to find the new output. Plot the first ten terms of your result. (3 marks)

5a). A program segment to determine the square root of a number reads as follows:

```
message := "Error - cannot take square root of a negative number";
IF x ≥ 0 THEN y := SQRT(x) ELSE y := message;
Writeln y;
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Given $x \in \mathbf{R}$,

i) define the sets A and B such that the cartesian product $A \times B$ gives rise to the relation R as described in the program segment. (3 marks)

ii) Is the relation a function?. Give your reasons. (2 marks)

5b). Given $A = \{1,2,3,4\}$ and the function $f: A \to A = \{(1,2),(2,4),(3,1),(4,3)\}$, show that $f \circ f \circ f \circ f = f^4 = I_A$. (5 marks)

Concordia University

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COEN 231/2 U Fall 1997
Discrete Mathematics

Final Examination

Value: 65% of final grade
Time: 3 hours
Total marks: 65

Materials allowed: Two 8 and 1/2 by 11 crib sheets (both sides)
Any inanimate computing aids you can manage to carry in.

Note: It is not necessary to work out factorials on your calculator. Doing so will not give you extra marks.

1a). In how many ways can a particle move in the xy plane from the origin to the point (7,6) if each move is one of the following types:
(H): $(x,y) \rightarrow (x+1,y)$; (V): $(x,y) \rightarrow (x,y+1)$? (4 marks)

1b). What is number of ways of distributing 12 identical coins among five containers given that each container must have a least one coin? (3 marks)

1c) What is the number of ways of arranging 12 distinct books among five shelves? The order of the books on a shelf is not important. (3 marks)

2a). Using truth tables, prove the inference rule:
$\{[p\wedge(p\rightarrow q)]\rightarrow q\}\Rightarrow T_0$. (5 marks)

2b). Using the laws of logic (not truth tables) prove that:
$(a\wedge b)\rightarrow c\Leftrightarrow a\rightarrow(b\rightarrow c)\Leftrightarrow b\rightarrow(a\rightarrow c)$. (5 marks)

2c). Prove the following argument:

$p\rightarrow q$
$q\wedge r\rightarrow s$
r
$t\rightarrow\neg s$

$\therefore p\rightarrow\neg t$ (5 marks)

2d). The universe consists of all people. All HiTech employees who write programs for the Internet must know Java. Albert works for HiTech but doesn't know Java. We conclude that Albert doesn't write programs for the Internet.

i) Establish the primitive open statements (1 mark)

ii) Set up the logical argument (1 mark)

iii) Prove the argument. (3 marks)

3a). Prove by induction that:

$$S_n = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2; n \in Z^+.$$ (5 marks)

3b) Given A = {1,2,3,4,5,6,7,8,9},

i) How many subsets of A contain 5 elements? (2 marks)

ii) How many 5 element subsets of A contain the elements 3 and 5? (3 marks)

4a). Convolve x = {3,1,3,1,3,1,..} and h = {0.25, 0.25, 0.25, 0.25, 0, 0,...}. Plot the first ten terms of your result. (5 marks)

4b). Solve the following first-order difference equation.

$y_n - ay_{n-1} = 1 - b^n$. (5 marks)

4c). Solve the following second-order difference equation.

$y_n - 0.9\sqrt{3}y_{n-1} + 0.81y_{n-2} = \varepsilon_{n-1}$ (5 marks)

5a). Given the Cartesian product **Z**×**Z** and the relation

R = {(x,y)| x ε **Z** , y ε **Z**, y = x^2 +1},

i) Say why the relation is a function? (2 marks)

ii) Identify the domain, the co-domain and the range of the function (3 marks)

5b). Given A = {a,b,c,d},

i) Construct one possible bijective functional mapping f: A→A. (2 marks)

ii) For your particular f, show in tabular format, that $f^4 = I_A$. (2 marks)

iii) How many possible bijective functions f: A→A are there? (1 mark)

Bonus marks – beyond the total of 65 for the exam. <u>Do not attempt this more difficult question until you have reviewed and are satisfied with your answers to all other questions.</u>

Given A = {a,b,c,d} and **all possible** bijective functions f: A→A, show that

∀f [$f^4 = I_A$.] WRONG!! (5 marks)